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ABSTRACT

This booklet is a collection of puzzles, investigations, and games. They are designed to be used with large objects such as tins or stones and diagrams marked on the ground. The children are to be encouraged to use an experimental, trial-and-error approach at first, and then develop methods of solution. (MNS)

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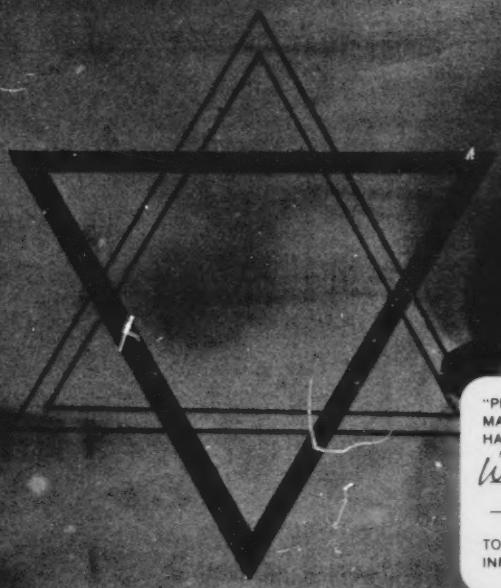
MORE PEBBLE PUZZLES



CAN YOU PLACE 6 STONES SO THAT THERE ARE 2 ON EACH LINE AND 3 ON EACH LARGE TRIANGLE?

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HOW TO USE THIS BOOK

First try the puzzles and investigations yourself. Then choose those you think are suitable for your class. Do one or two examples with the children not on the board but using large objects such as tins or stones and a diagram marked on the ground. Group the children so they can all see. Tell them the rules of the puzzle and go through a couple of examples. Next let all the children try themselves, using their own pile of stones and a diagram drawn on a scrap piece of paper or in their exercise books.

Encourage the children to use an experimental approach at first. Many of the simple puzzles can be solved by trial and error. Soon the children will discover that if they think first they can limit the number of trials they have to make. Encourage them to develop methods of solution for themselves. This is the main aim of this collection of puzzles. The answers are not important or valuable in themselves. What is important is the development of the childs' ability to solve problems. This comes through searching and solving on their own. The reason for using puzzles is that a good puzzle is fun, is easy to understand and such that you know yourself when the answer is correct. This gives reinforcement to the method of solution straight away. Most children will want you to confirm that their answer is correct but try and develop their own critical ability to judge between a right and wrong solution.

When the children have had some success at solving puzzles of a certain kind ask them to make up similar puzzles of their own. This introduce a creative element into mathematics lessons, an element which is often absent. It also develops the concept of 'reversal'. The best way to make up a new puzzle is to start with the answer and work backwards. Through making up puzzles the child will become more aware of the mechanics of the puzzle, when and how it works. For higher level students this is further encouraged through the investigations which follow each set of puzzles in Section A. Here the student is challenged to discover and generalise the patterns and structures involved and to express these in algebraic form. At this level he may notice similarities between puzzles which at first appeared to have nothing in common. Perhaps a completely new type of puzzle will suggest itself, by dressing up the bare bones of the structure in a new way or by varying the original conditions.

One last point. These puzzles are meant to motivate the children to find their own solutions. It is up to you to present the puzzles as a challenge to the chidlren and to stimulate their eagerness to solve them. It is up to you to make sure that the children are successful enough to want to do more.

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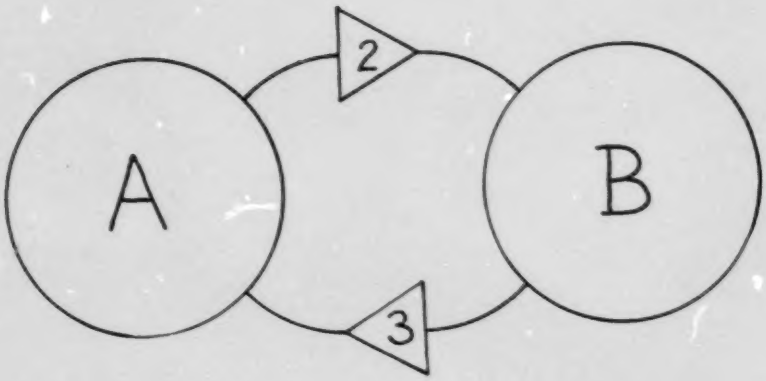
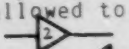
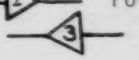
PUZZLES AND INVESTIGATIONS

ROUTES



Puzzle 1

Put 5 stones in A. Can you get all the stones from A to B? You are allowed to move stones in groups of 2 along the  route and stones in groups of 3 along the  route in the direction shown by the arrow.

Puzzle 2

Now try starting with 7 stones in A. How many moves does it take to get them all to B.

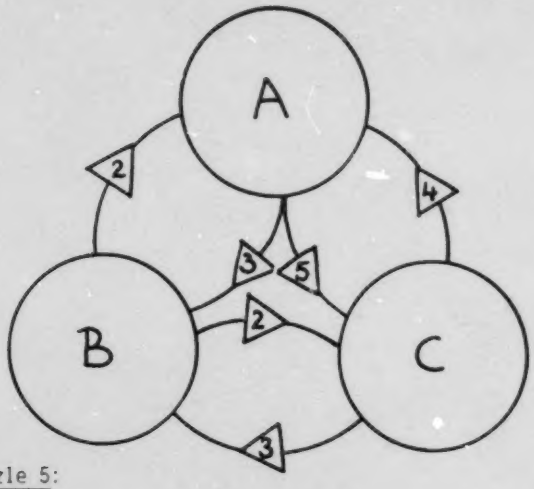
Puzzle 3

Now start with 5 stones in B. Can you get them all to A? What is the fewest number of moves.

Puzzle 4

Now start with 7 stones in B. Can you move them all to A?

What is the fewest number of moves?



Puzzle 5:

Put 7 stones in A. Try and get all the stones to C in 5 moves. You are only allowed to move stones as shown by the numbers and arrows.

Puzzle 6:

Can you get 7 stones from C to A in 6 moves?

Puzzle 7:

Can you get 7 stones from C to A in 7 moves?

Puzzle 8:

Can you get 8 stones from A to C in 6 moves?

Puzzle 9:

Can you get 9 stones from A to C in 6 moves?

Puzzle 10

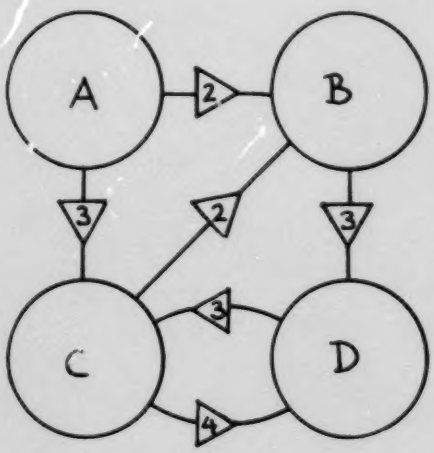
(1) Start with 5 stones in A and get them all to D in 11 moves.

(2) Move 6 stones from A to D. (5 moves)

(3) Move 7 stones from A to D. (8 moves)

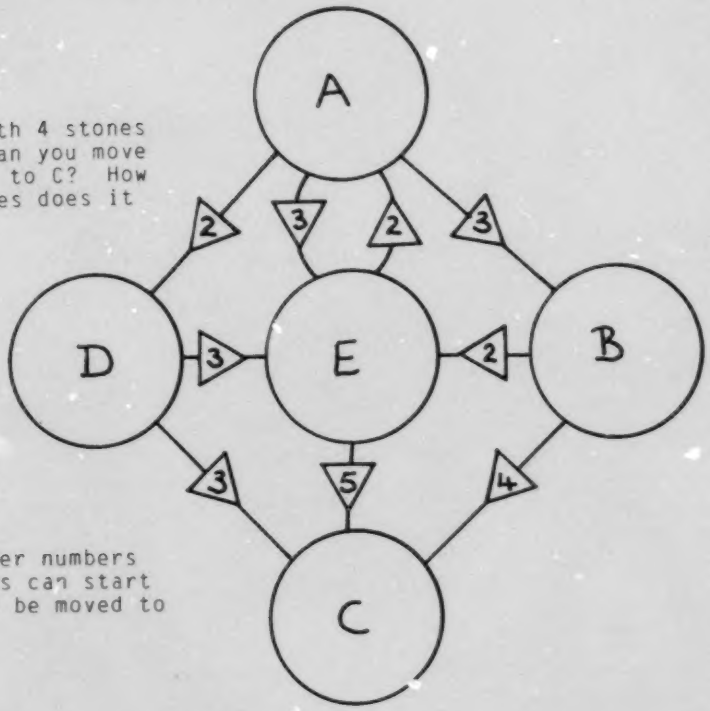
(4) Move 8 stones from A to D. (11 moves)

(5) Move 9 stones from A to D. (7 moves)



Harder Puzzle

(1) Start with 4 stones at A. Can you move them all to C? How many moves does it take?



(2) What other numbers of stones can start at A and be moved to C?

Investigation

(1) Use the diagram from the first puzzle 9. If we start with 4 stones in A how many moves does it take to move them to B? Build up a table showing the number of stones starting in A and the number of moves it takes to get them to B. Can you see a pattern in the number of moves?

How many moves would 40 stones take?

How many moves would 41 stones take?

(2) Now start with the stones in B and work out a similar table showing the number of moves. Look for a pattern.

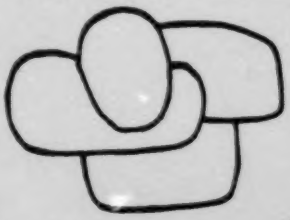
How many moves do you think 60 stones take? How about 61? and 62?

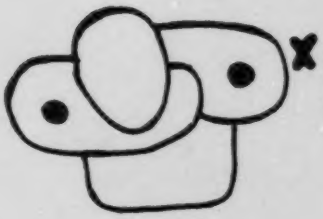
7

SPACE PUZZLES

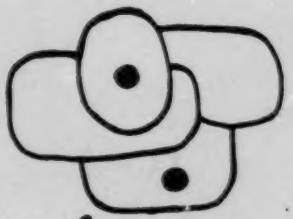
The rules for these puzzles are quite simple.
(1) You can only put one stone in a space.
(2) Stones must not be in spaces which touch.

Example: Put 2 stones on this diagram.





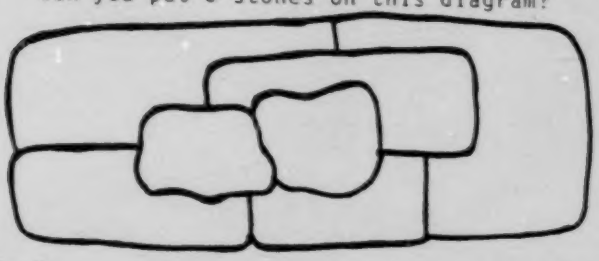
✘ Wrong because these two spaces touch.



✓ These spaces do not touch.

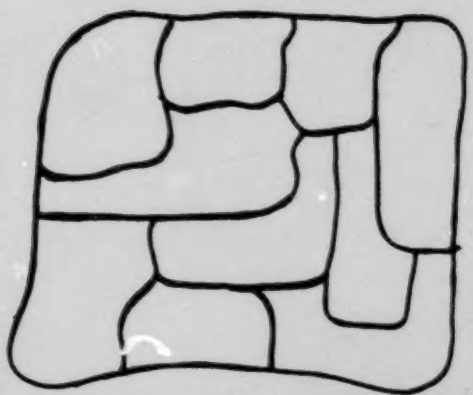
Puzzle 1

Can you put 3 stones on this diagram?



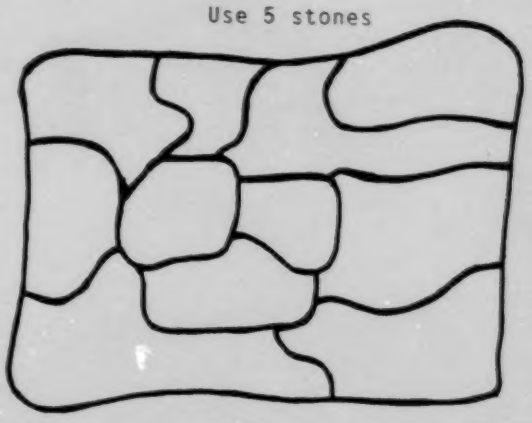
Puzzle 2

Use 4 stones



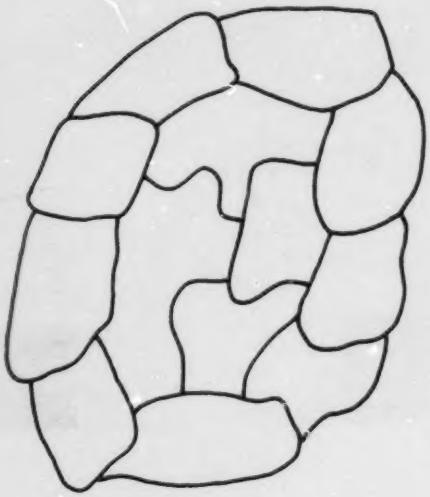
Puzzle 3

Use 5 stones



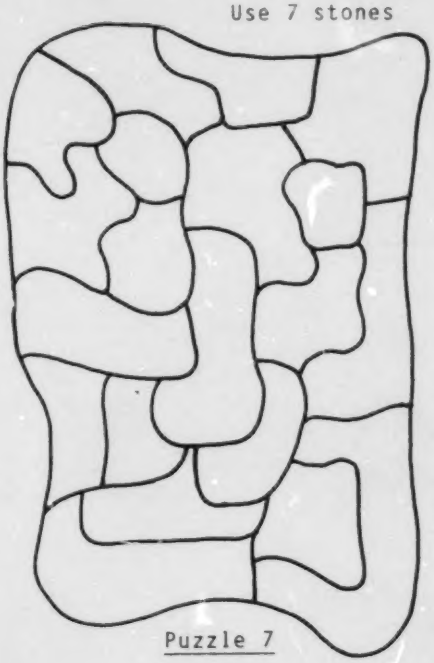
Puzzle 4

Use 5 stones



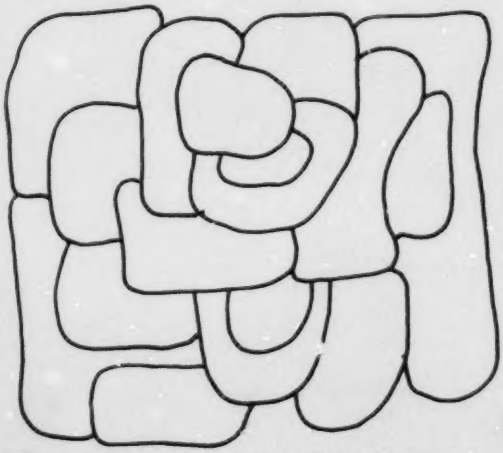
Puzzle 5

Use 7 stones



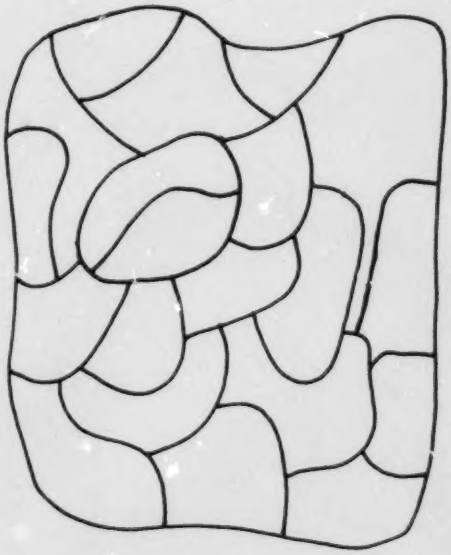
Puzzle 6

Use 8 stones



Puzzle 7

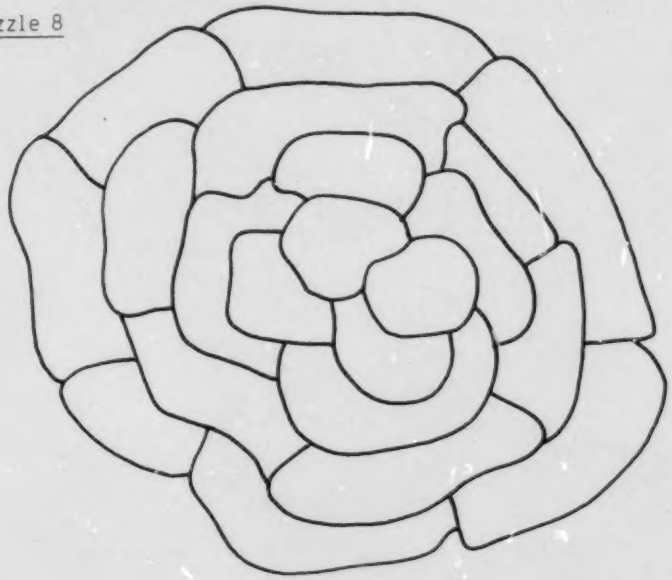
Use 9 stones



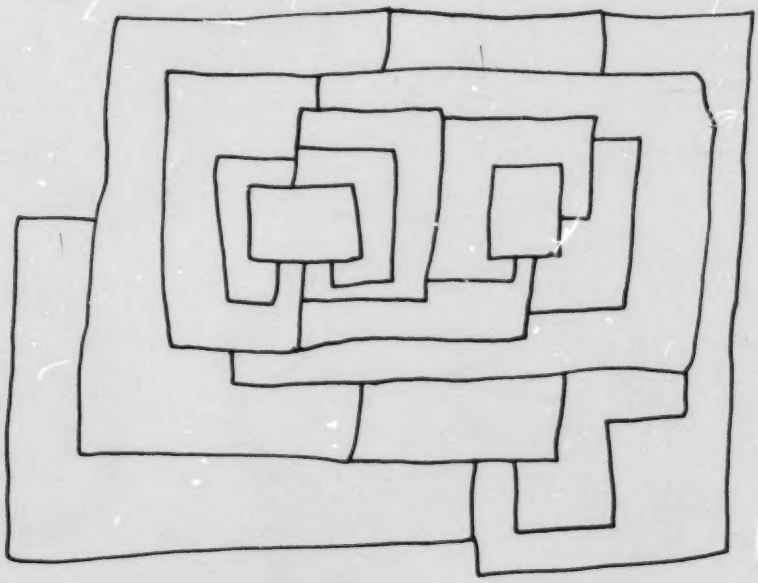
Make up some puzzles like these.

What is the greatest number of stones you can place on each of these diagrams? Remember that spaces with stones in them must not touch!

Puzzle 8

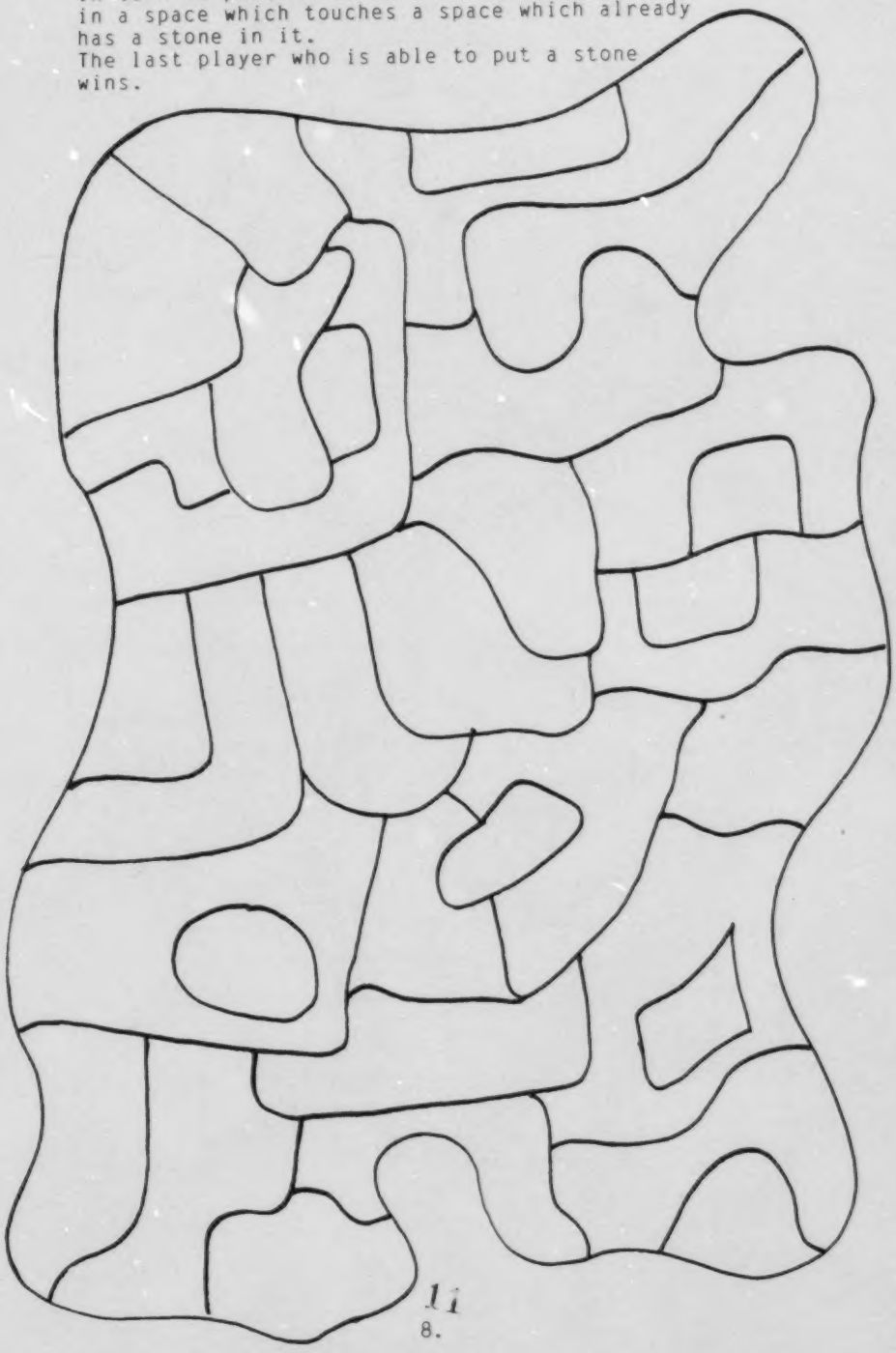


Puzzle 9

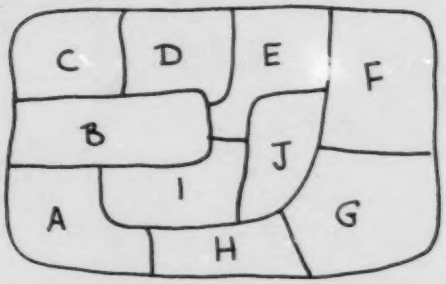


Space Game - 2 players.

Each player has a pile of stones. Take it in turn to place a stone in a space but not in a space which touches a space which already has a stone in it.
The last player who is able to put a stone wins.



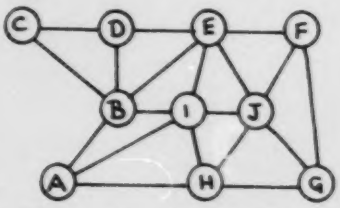
Space Investigation



1. This is a board for a Space Game. The first player puts his stone in A. Where should you go now so that you will be sure to win?

2. If you were first player where would you put your stone so that you would be certain to win?

Network



3. Here is a new game for 2 players. Each player has a pile of stones. In turn each player places a stone on a circle but no player may put a stone on a circle which is joined by a line to a circle with a stone already in it.

For example, if there is a stone in A, you may not put a stone in B, I or H.

The last player to place a stone is the winner. Can you work out where the first player should go in order to win?

4. What is the connection between this game and the Space Game?

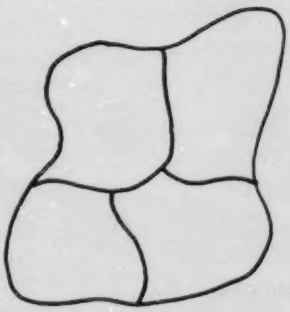
Harder Space Puzzles

Puzzle 1

Place 7 stones so that:

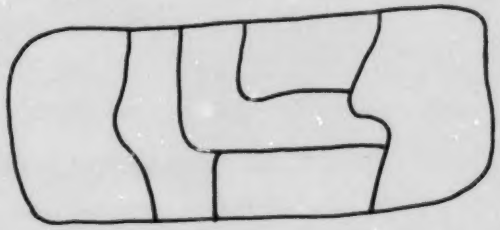
(1) each space has at least one stone,

(2) no two spaces with the same number of stones are touching.



Puzzle 2

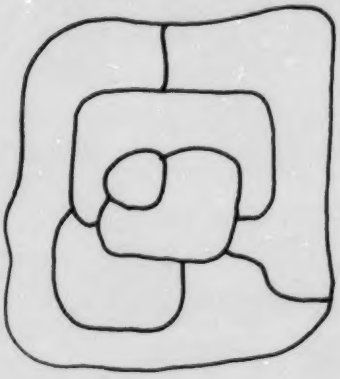
Using the same rules place 10 stones on this diagram.



Use the same two rules in each of these puzzles:

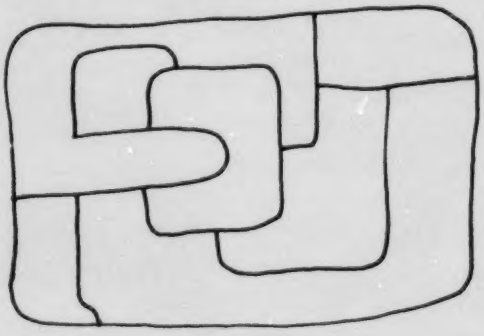
Puzzle 3

Use 12 stones.



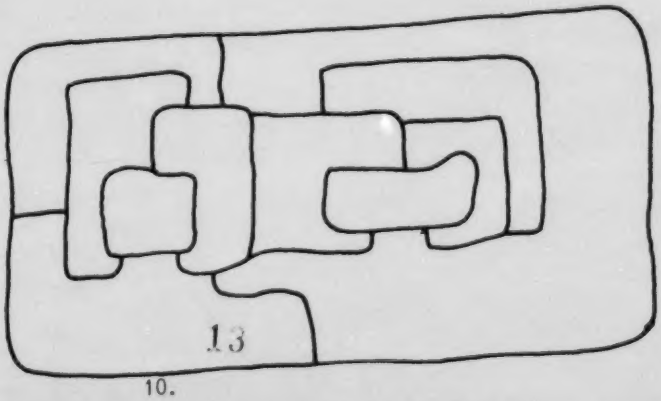
Puzzle 4

Use 14 stones.



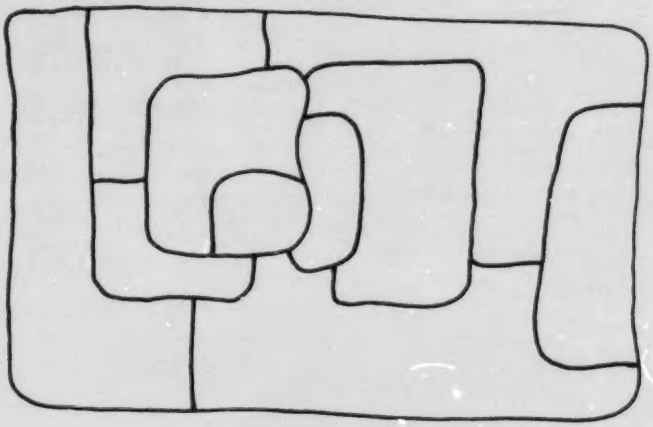
Puzzle 5

Use 19 stones.



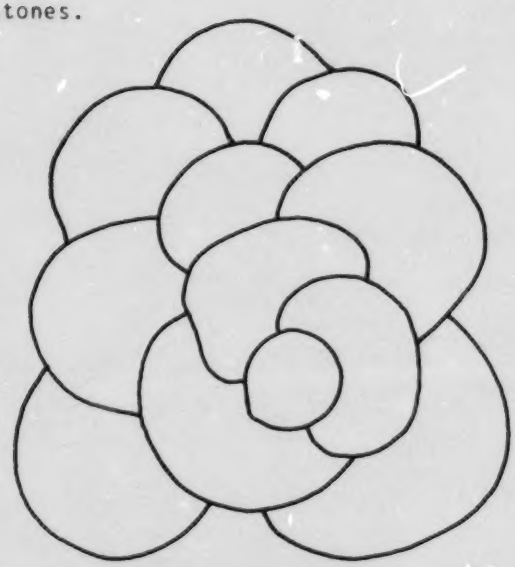
Puzzle 6

Use 20 stones.



Puzzle 7

Use 23 stones.



Draw spaces of your own and try and discover the smallest number of stones you need to fill every space according to the two rules.

INVESTIGATION

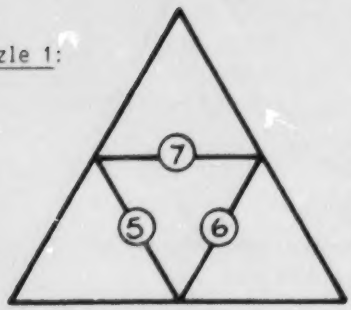
1. Investigate diagrams made with 3 spaces only. Can you draw a diagram that uses less than 4 stones? Can you draw a diagram that needs more than 6?

2. Now investigate diagrams made with 4 spaces. Can you draw the diagram that needs the least stones? And the most stones?

3. Now try 5 spaces. Find diagrams which need the least stones. Can you draw a diagram that needs more than 11 stones?

TETRARCHS

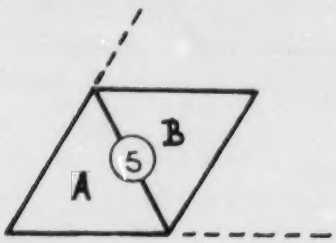
Puzzle 1:



10 stones

Example.

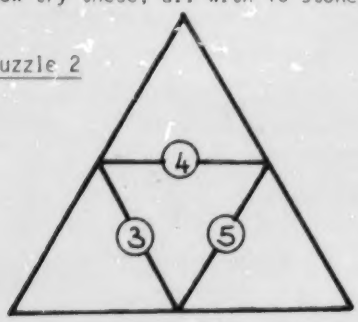
Take 10 stones. Put them in the four triangles so that the total in every two joined triangles is equal to the number in the circle.



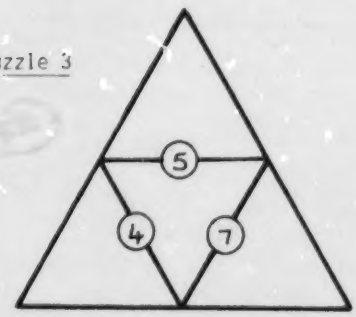
So there could be 1 in A and 4 in B as 1 + 4 = 5, or 2 in A and 3 in B etc.

Now try these, all with 10 stones.

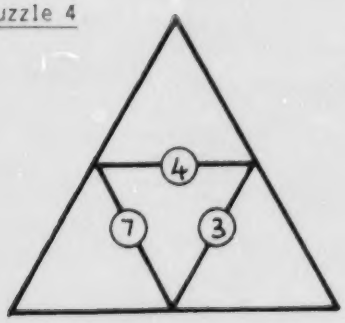
Puzzle 2



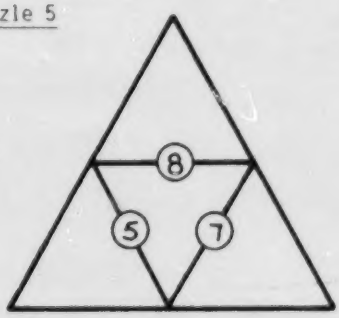
Puzzle 3



Puzzle 4

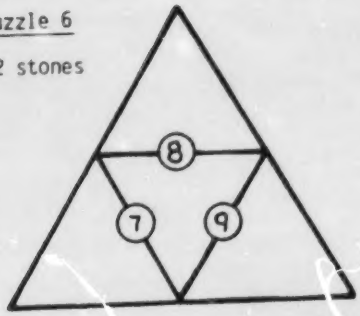


Puzzle 5



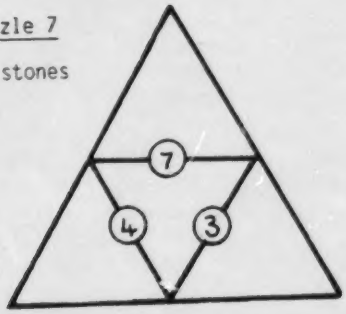
Puzzle 6

12 stones



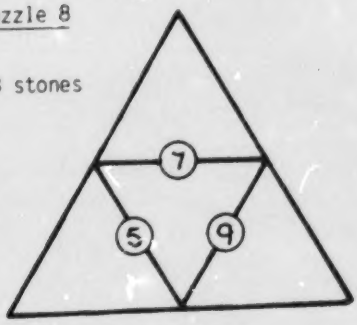
Puzzle 7

12 stones



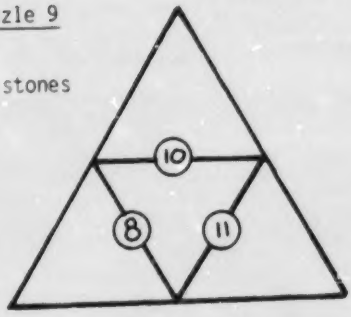
Puzzle 8

13 stones



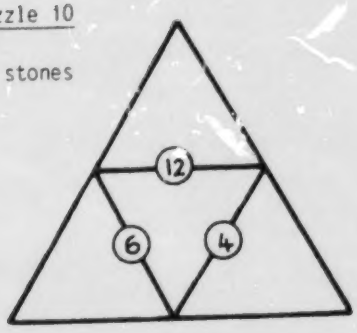
Puzzle 9

15 stones



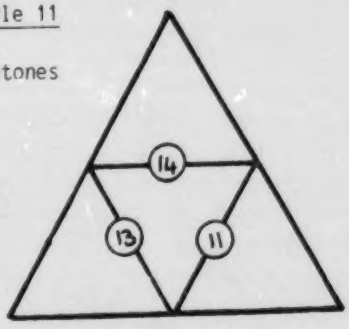
Puzzle 10

18 stones



Puzzle 11

20 stones



Now try making up some of your own. Put the stones, record the numbers that go in the circles, then remove stones and ask a friend to solve it!

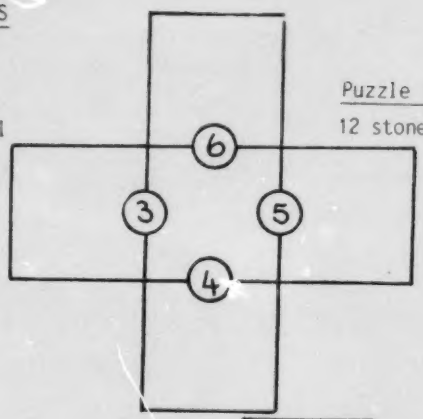
17

CROSSES

Here are some more similar puzzles. Now there are 5 squares into which you must put the stones. The total for each pair of joined squares is in the circle.

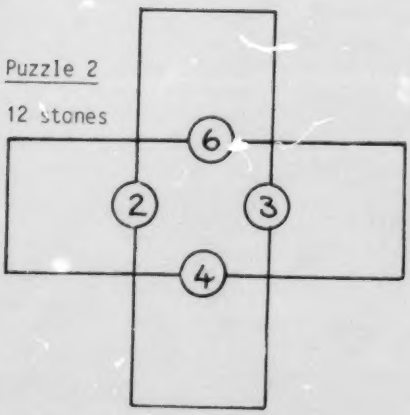
Puzzle 1.

12 stones



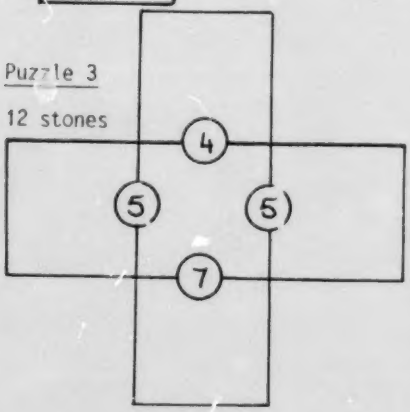
Puzzle 2

12 stones



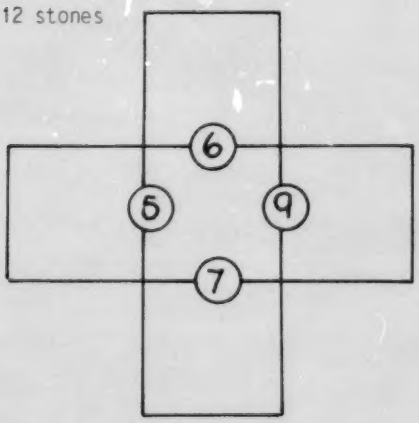
Puzzle 3

12 stones



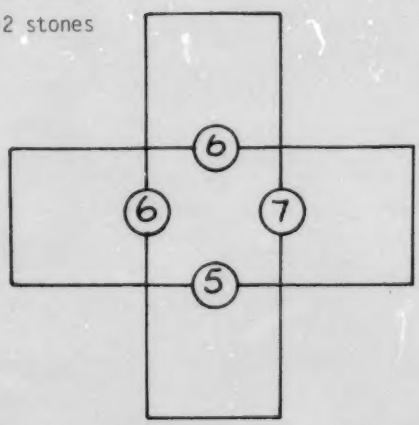
Puzzle 4

12 stones



Puzzle 5

12 stones

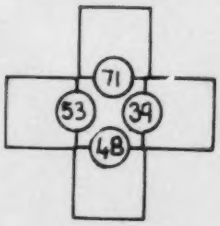


(All of these puzzles are also possible with 15 stones! Can you find solutions).

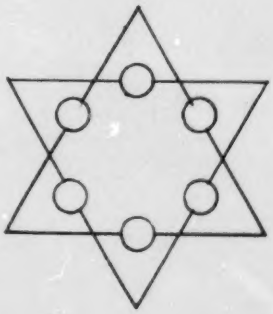
INVESTIGATION.

(1) Can you solve this Tetrarch Puzzle? It is not practical to use a trial and error method, so try and work out a general way to solve all Tetrarch puzzles.

(2) Now try and solve this Cross Puzzle which uses 100 stones. Can you find a general method of solution for all cross puzzles?



(3) If we invented Star Puzzles which had the same rules as Tetrarchs and crosses, can you see what rules we should follow when placing numbers in the circles?

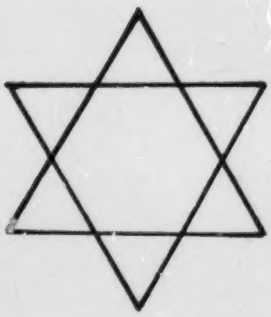


STARS

TRIANGLE STAR

Take 6 stones
Place them so there are:

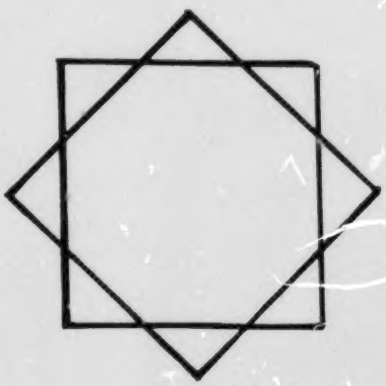
(1) 2 stones on each line and 3 stones on each large triangle



(2) 2 stones on each line and 6 stones on each triangle.

(3) 2 stones on each line and 5 Stones on each triangle.

(4) 2 stones on each line and 4 stones on each triangle.



SQUARE STAR

Take 4 stones.
Place them so that there are

(1) 1 stone on each line and 2 stones on each square.

(2) 1 stone on each line and 4 stones on each square

(3) 1 stone on each line and 3 stones on each square.

Take 8 stones. Place them so that there are 2 stones on each line and

(1) 4 stones on each square. (2) 5 on each square.

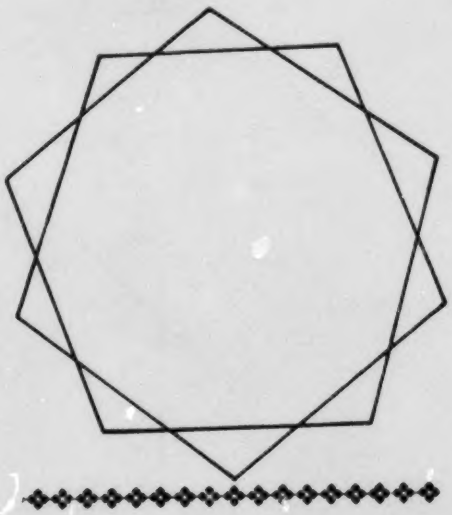
(3) 6 stones on each square. (4) 7 on each square.

(5) 8 stones on each square.

PENTAGONAL STARS

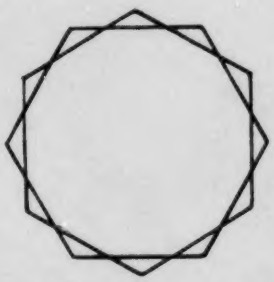
Take 10 stones. Place them on the lines so that there are 2 stones on each line and

(1) 5 stones on each pentagon.
(2) 6 stones on each pentagon.
(3) 7 stones on each pentagon.
(4) 8 stones on each pentagon.
(5) 9 stones on each pentagon.
(6) 10 stones on each pentagon.



Investigation

(1) What similar puzzles can be made up on the hexagonal star? With 6 stones.
With 12 stones.



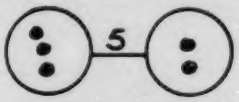
(2) Can you work out a tactic for solving these star puzzles?

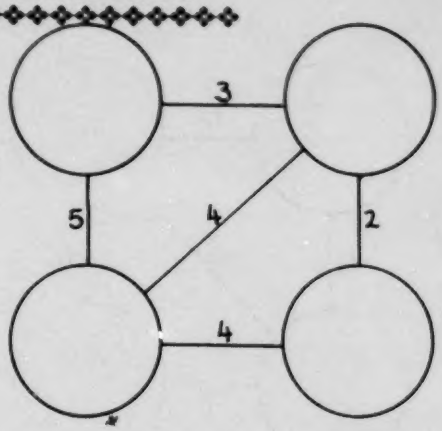
SQUARE PUZZLES

Puzzle 1:

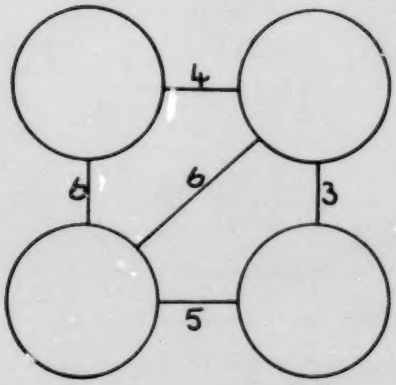
Take 7 stones. Put them in the circles so that the sum of two circles is equal to the number on the line.

Example.

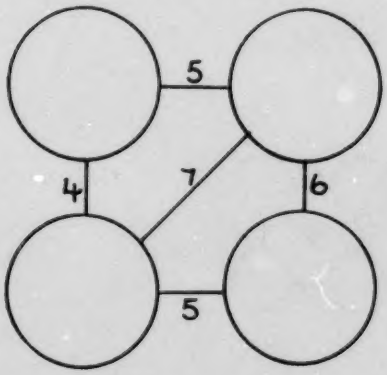




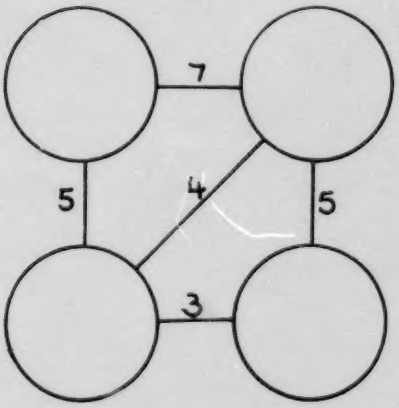
Puzzle 2: 9 stones.



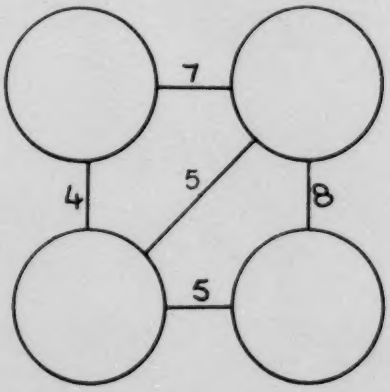
Puzzle 3: 10 stones



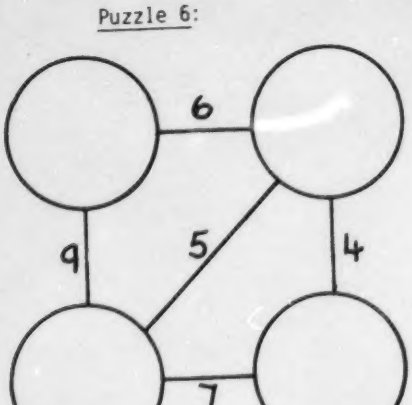
Puzzle 4: 10 stones



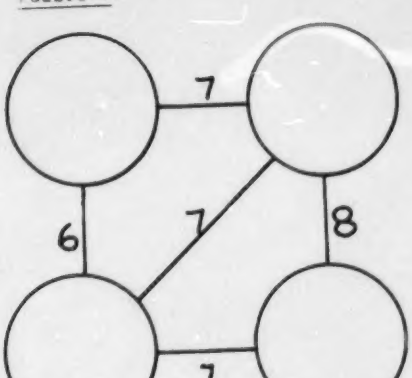
Puzzle 5: 12 stones.



Puzzle 6:

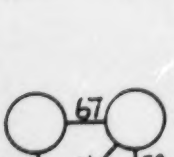


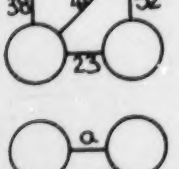
Puzzle 7:



Can you do these without being told how many stones to use?
Can you make up some puzzles like these?

INVESTIGATION.



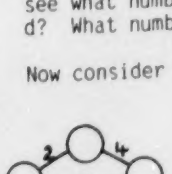


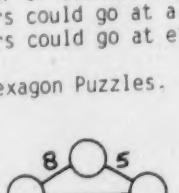
(1) How can you find out the number of stones to be used in Puzzles 6 and 7?

(2) Can you find a method to solve all problems like these no matter how big the numbers? For example this one.

(3) Given any number of stones, say 10, can you see what numbers could go at a and b, c and d? What numbers could go at e?

(4) Now consider Hexagon Puzzles.



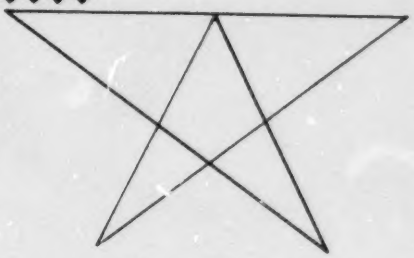


Can you find a method of solving them?

POLYGRAMS

Pentagram Puzzle

Place two stones on the diagram so that there is one stone on each straight line.



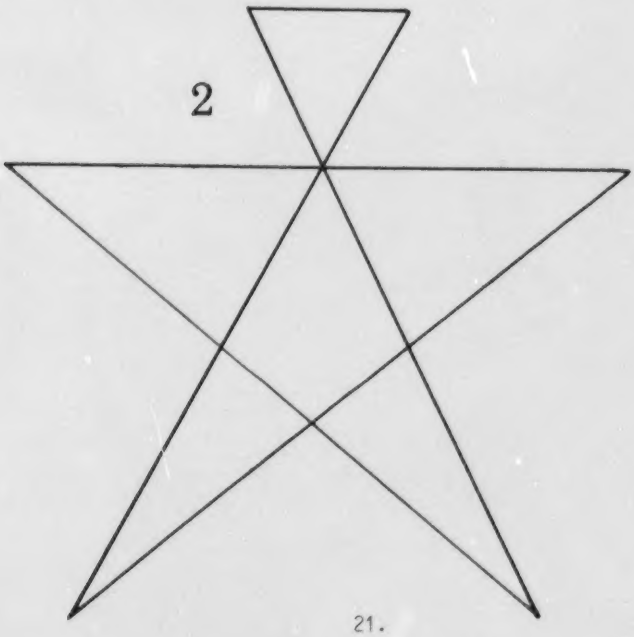
Hexagram Puzzles

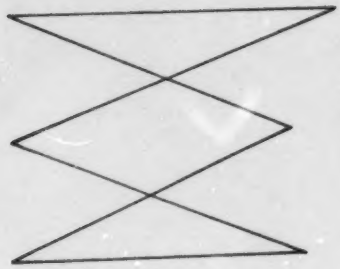
Place 2 or 3 stones on the diagram so that there is only 1 stone on each line. Stones must only be placed at vertices:

(One of these puzzles is impossible; can you find which)

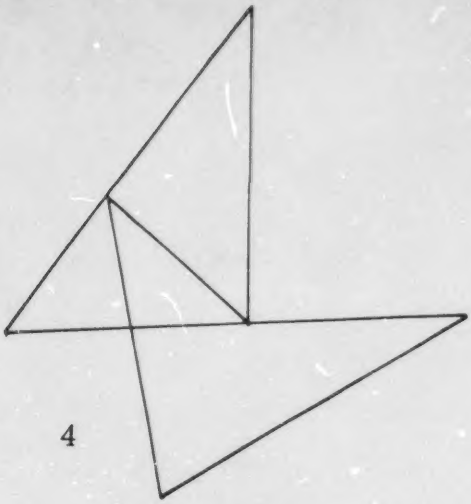
1

2

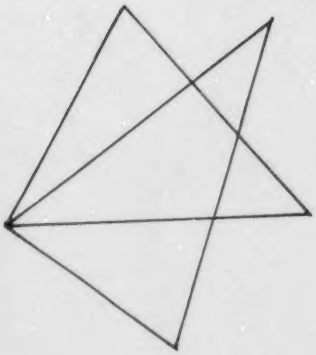




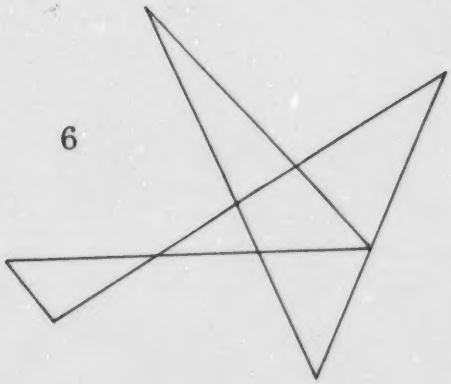
3



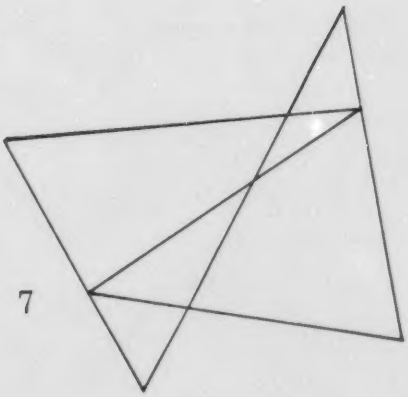
4



5



6

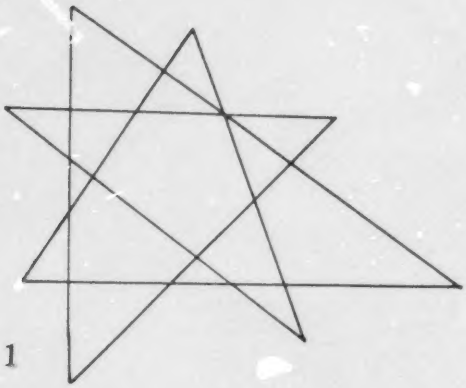


7

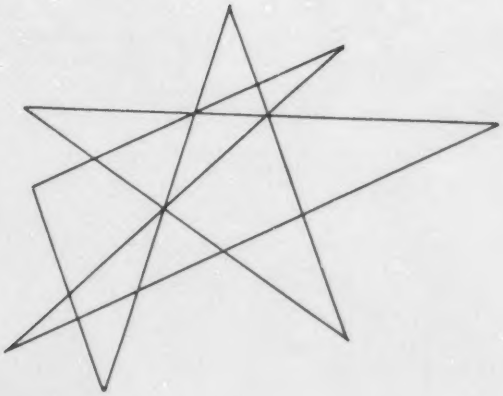
Polygrams

Find the least number of stones that can be placed on each diagram so that there is just one stone on each line. Stones must be placed on vertices.

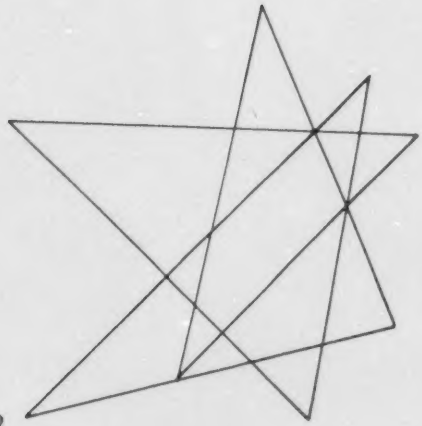
Octagrams



1

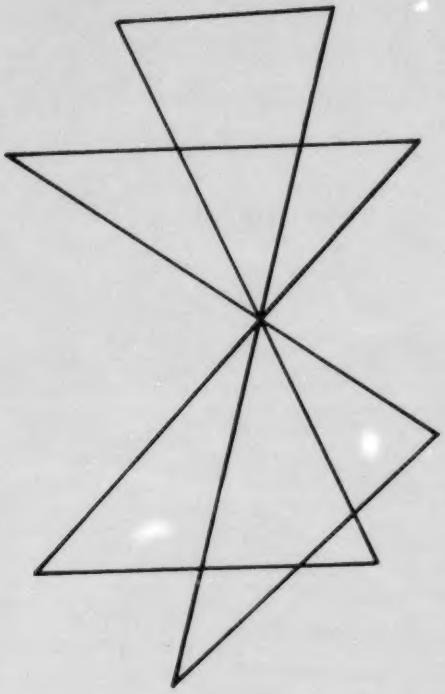


2

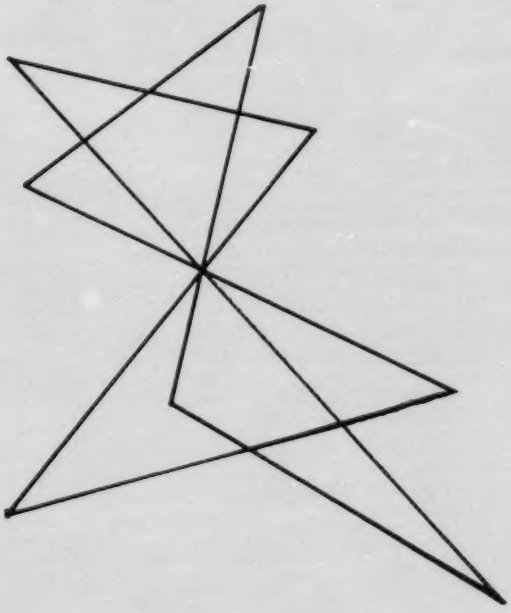


3

4

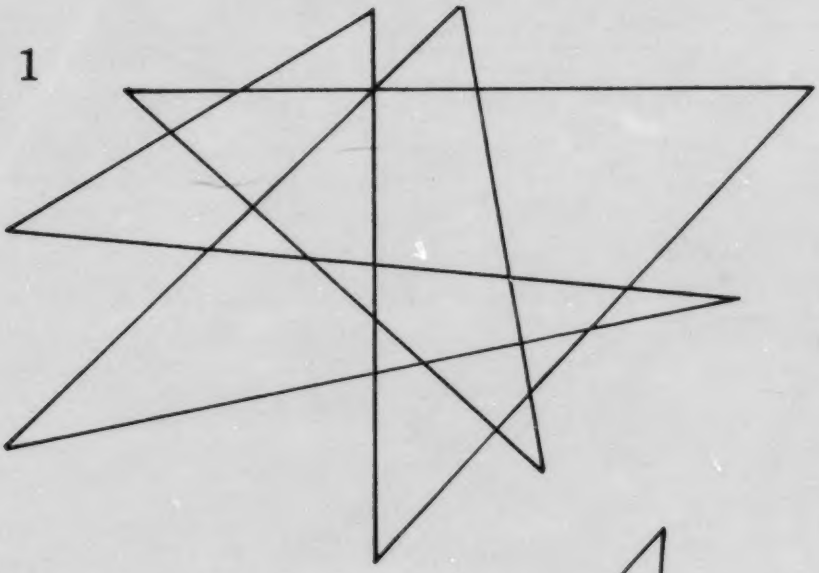


5

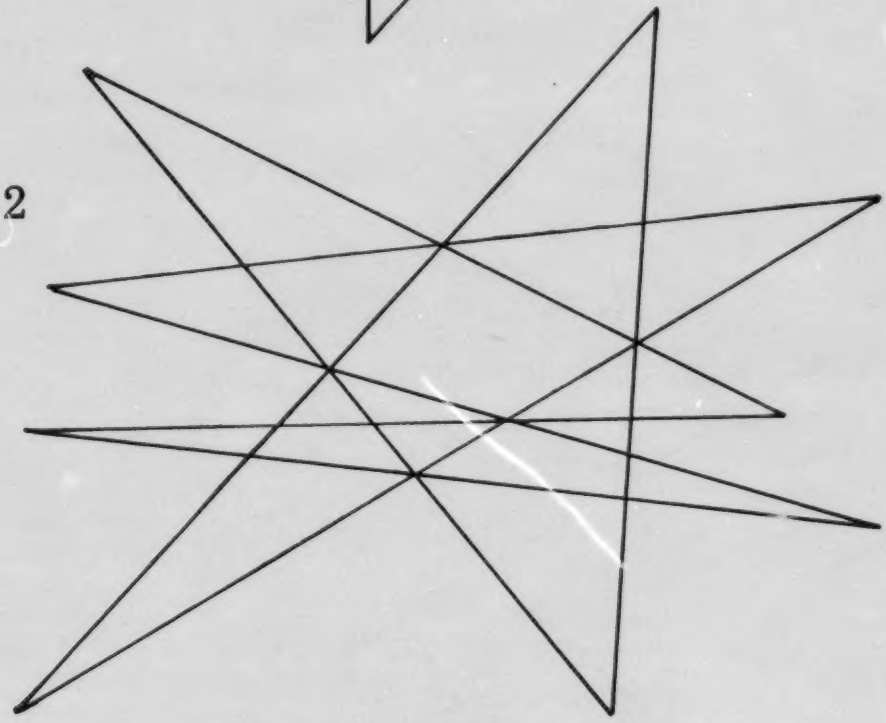


Nonograms

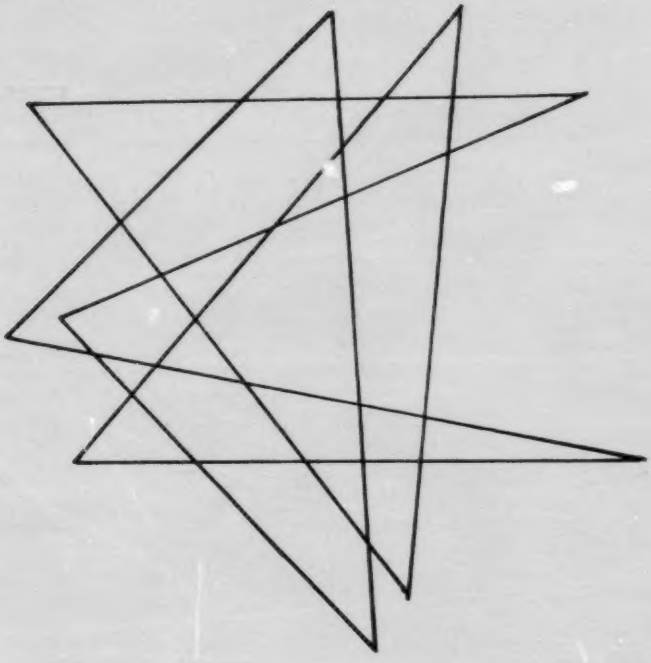
1



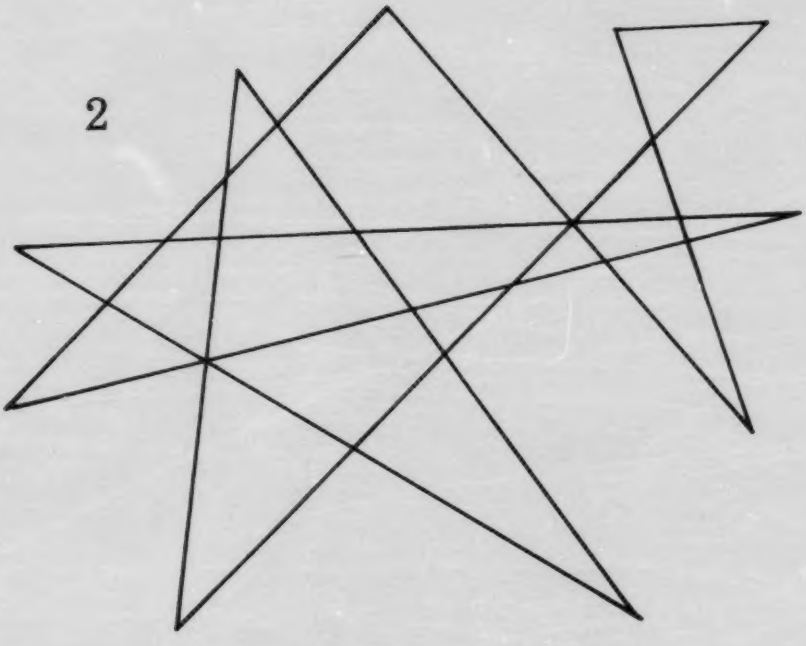
2



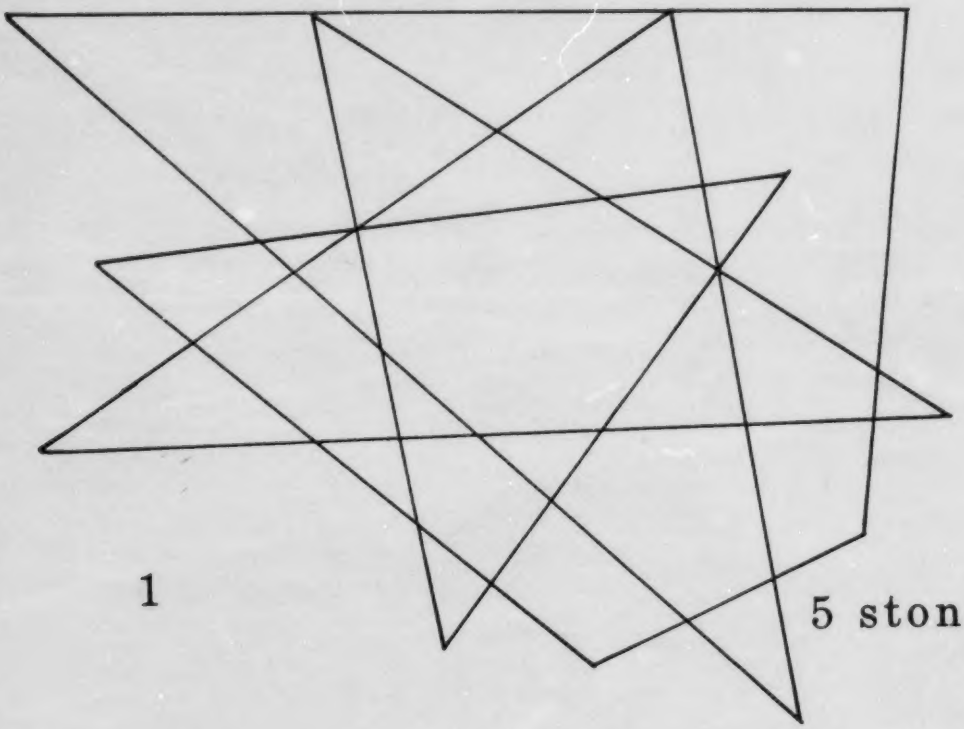
1



2

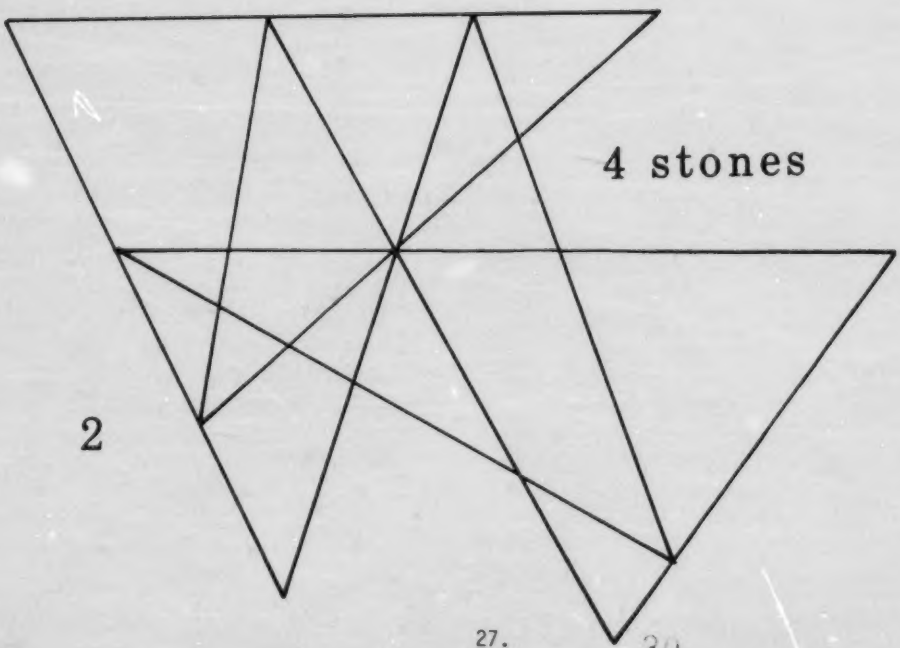


HARDER POLYGRAM PUZZLES



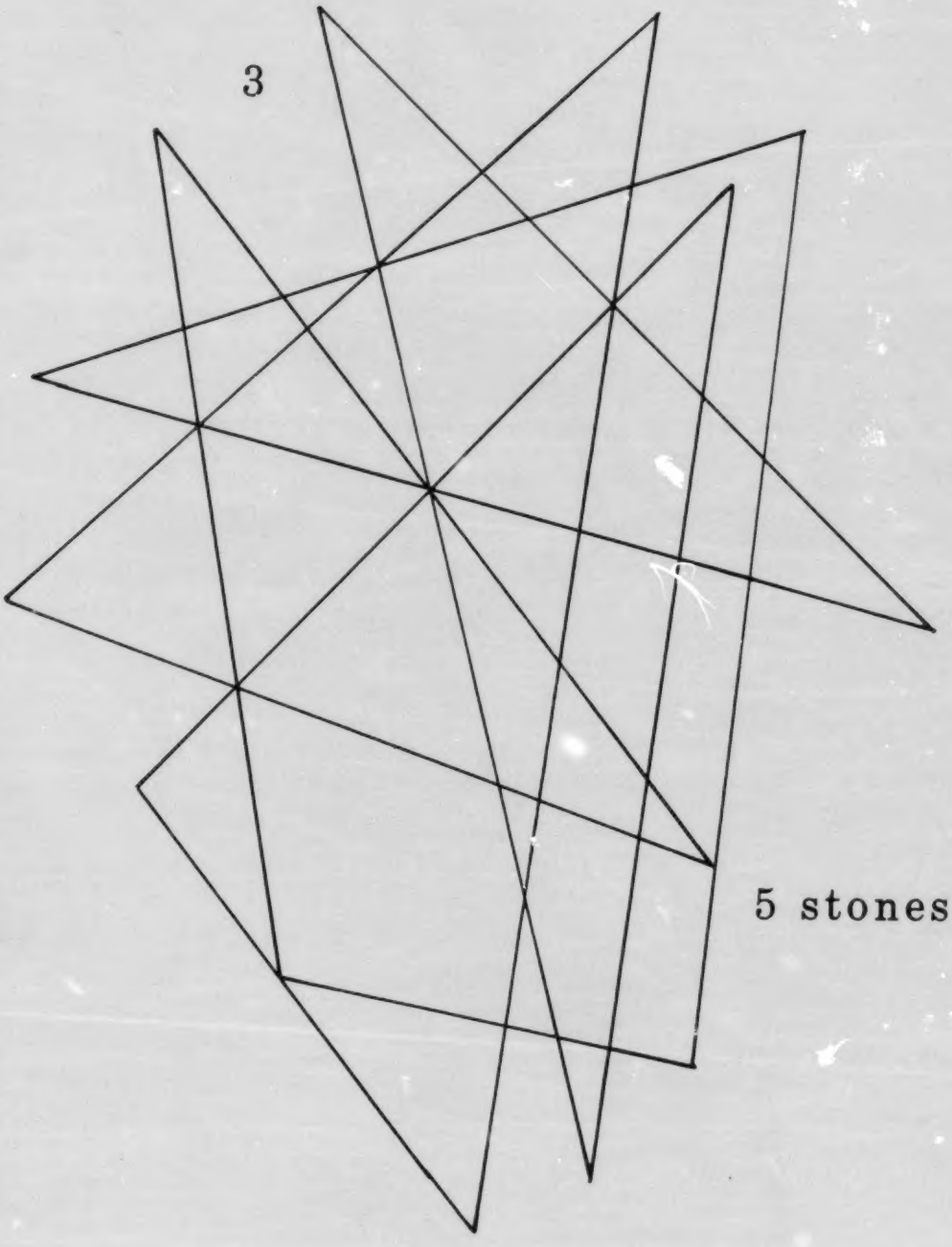
1

5 stones



4 stones

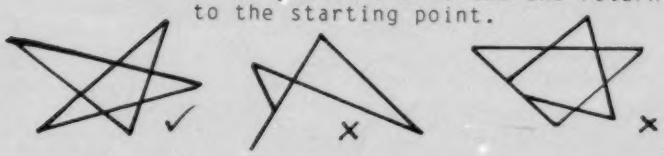
2



3i

INVESTIGATION

We will define a polygram as a figure made of straight lines and closed. That is; drawn in continous straight lines joined end to end and returning to the starting point.



1. Draw a TRIGRAM. (3 straight lines)
   Can you place stones on the vertices so there is just one on each line?

2. Investigate Quadrigrams (4 lines).
   How many different patterns can you make?
   Can you place stones on the vertices so that there is 1 on each line?

3. Investigate Pentagrams (5 lines) in the same way? Why are some possible and others not? Can you design both possible and impossible pentagram puzzles?

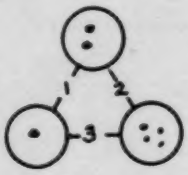
4. Can you generalise your findings to all polygrams?
   What tells you if a polygram puzzle is possible?
   How many stones are needed?

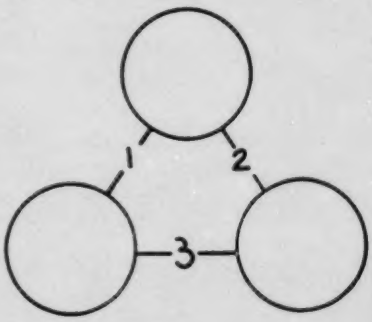
TRIANGLES
with a difference

Example

Take 7 stones. Place them in the circles so that the difference between the circles is the number written on the line.

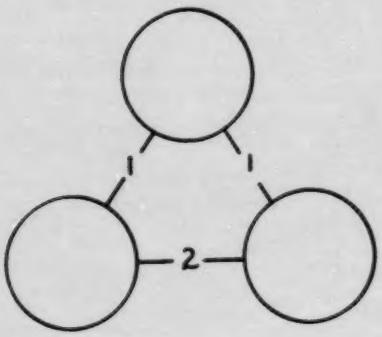
Answer





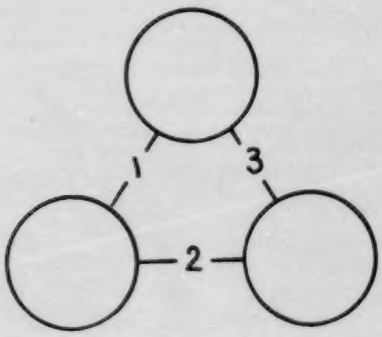
Puzzle 1

6 stones.



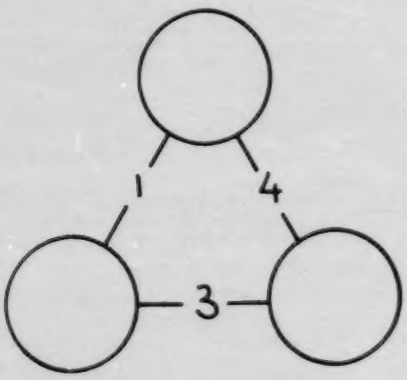
Puzzle 2

8 stones.



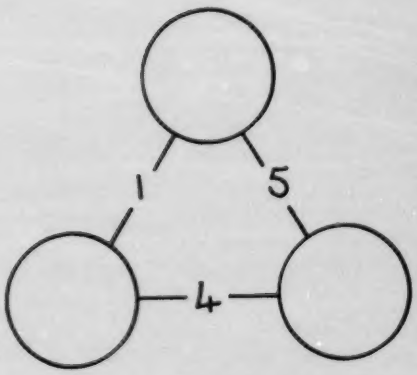
Puzzle 3

8 stones.



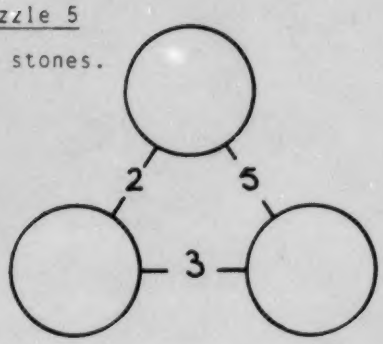
Puzzle 4

9 stones.



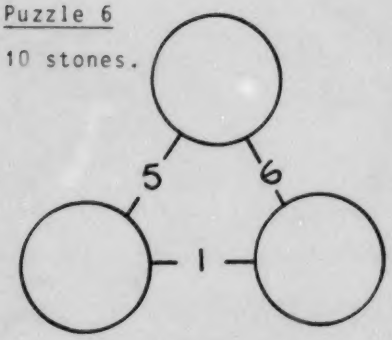
Puzzle 5

10 stones.



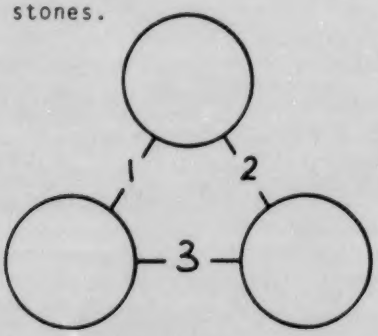
Puzzle 6

10 stones.



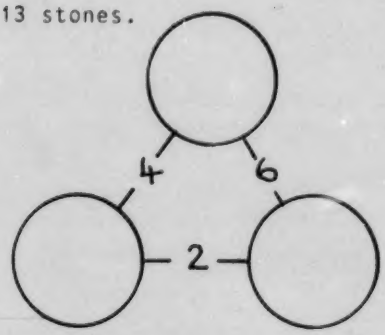
Puzzle 7

11 stones.



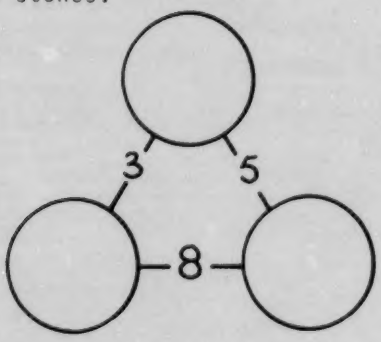
Puzzle 8

13 stones.



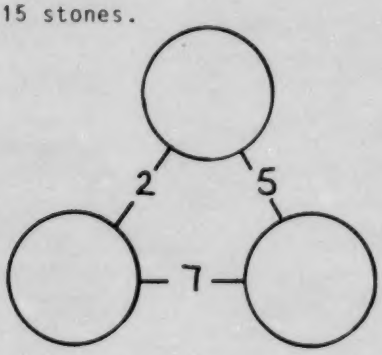
Puzzle 9

14 stones.



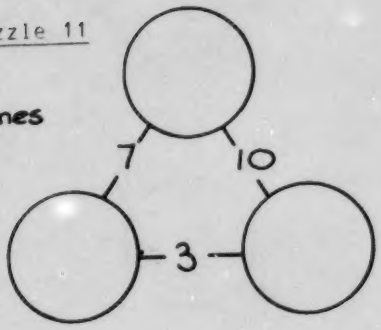
Puzzle 10

15 stones.



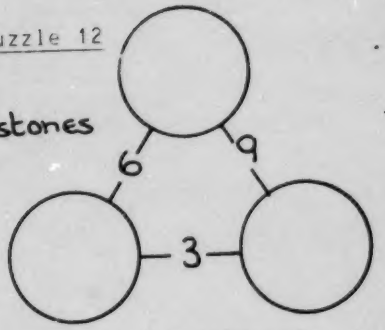
Puzzle 11

16 stones



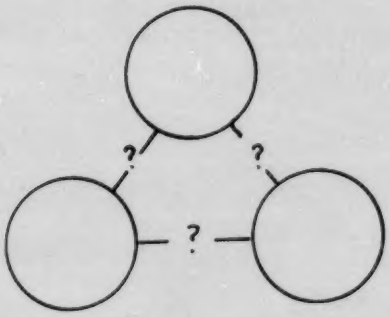
Puzzle 12

18 stones



Can you find 2 different solution to this puzzle?

INVESTIGATION



1. Using just 7 stones how many different puzzles can you make up?

2. Did you notice anything special about the numbers that go between the circles? There is a simple rule to be found.

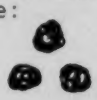
3. This is much harder: Can you find a general method of solving these puzzles, no matter how many stones are used?

4. In puzzle 12 there were two possible solutions. Can you discover any other puzzles which have two solutions?

CHANGE THE SHAPE

Puzzle 1

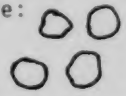
Place 3 stones on the dots to make this shape:



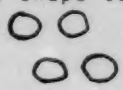
Move 1 stone and make this shape:

Puzzle 2

Place 4 stones to make this shape:

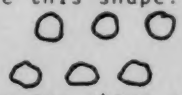


Move one stone and change the shape to:

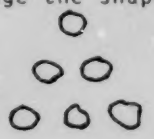


Puzzle 3

Place 6 stones to make this shape:

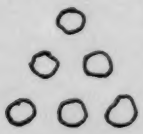


Move one stone and change the shape to:

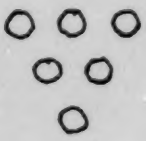


Puzzle 4

Place 6 stones in a triangle like this:

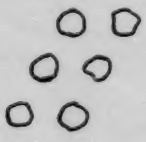


Move just two stones and change the shape to a triangle like this:

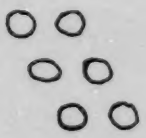


Puzzle 5

Make this with 6 stones

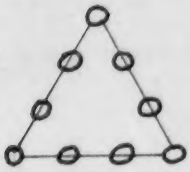


Move just 2 stones and change it to this:

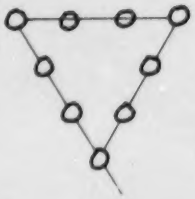


Puzzle 6

Take 9 stones and make this triangle.

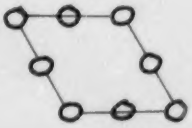


Move just 3 stones and change the shape to look like this.

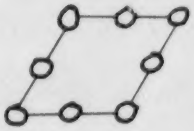


Puzzle 7

Take 8 stones and make this rhombus.

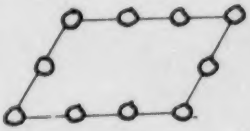


Move two stones and change it to this rhombus.

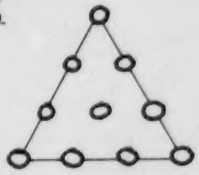


Puzzle 8

Take 10 stones and build a parallelogram like this:

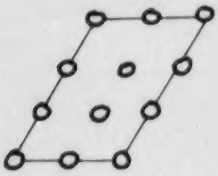


Move 3 stones and make a triangle like this:

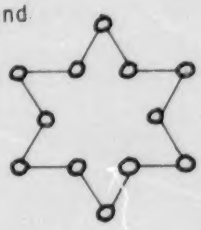


Puzzle 9

Take 12 stones and make this parallelogram.



Move 3 stones and make this star:



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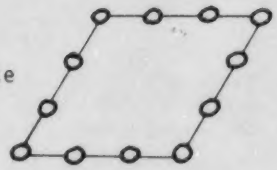
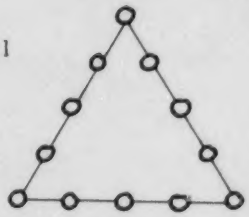
Puzzle 10

Use 12 stones to make a rhombus:



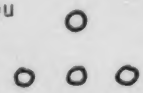
Move 3 stones and make an equilateral triangle.



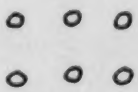
Change The Shape Investigation

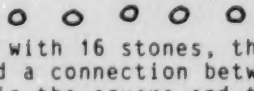
1. Start with 4 stones in a square pattern like this:

How many stones do you have to move to make this triangle?



2. What happens if you start with a square of 9 stones? How many moves does it take to make a triangle?

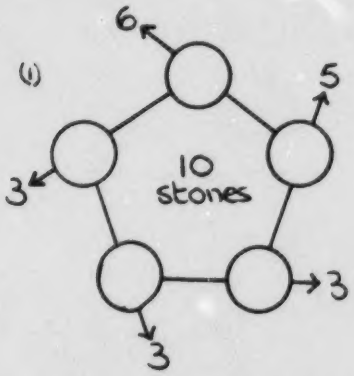




3. Now try with 16 stones, then 25, then 36. Can you find a connection between the number of stones in the square and the number you have to move to make a triangle (Hint: there is one rule for even numbers and one rule for odd number.)

PENTAGONAL PUZZLES

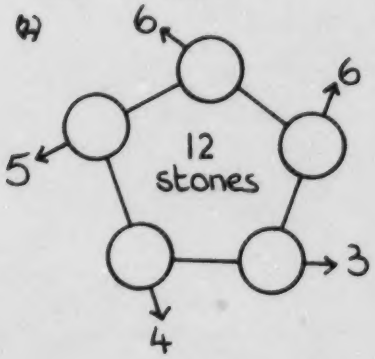
(1)



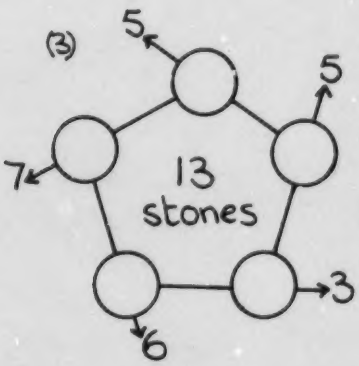
Take 10 stones. Place them in the circles so that the stones on each side add up to the number shown by the arrow.

Now try these:

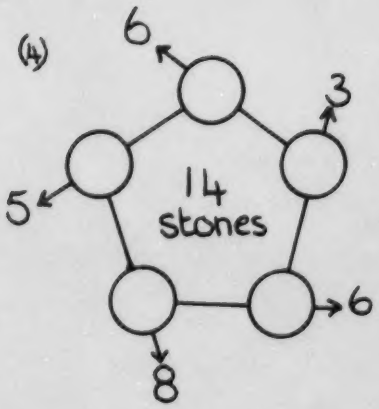
(2)



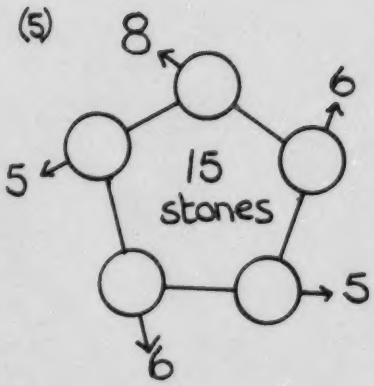
(3)



(4)



(5)



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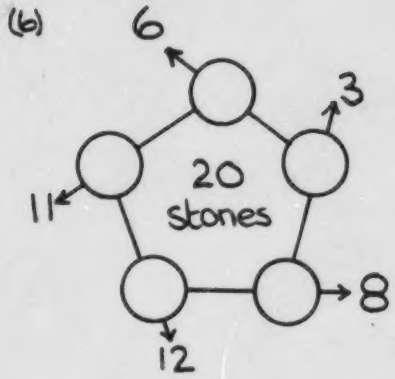
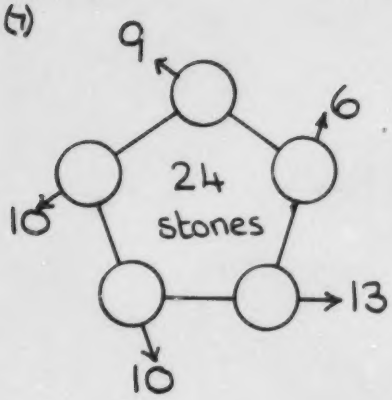
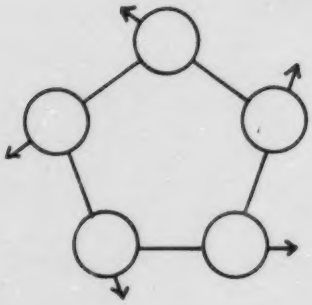
(6)



(7)



PENTAGONAL PUZZLES - Investigation



1. If we put 1,2,3,4,5, at the ends of the arrows would the puzzle be possible?

2. Is there a set of consecutive numbers that could be placed around the pentagon at the ends of the arrows?

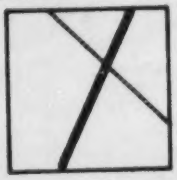
3. Could this set be placed in any order around the pentagon?

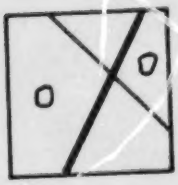
BARRIERS

The aim of these puzzles is to place the stones so that each 'barrier' or line divides the stones in half.

Example

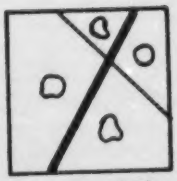
Use 2 stones on this diagram.





answer
with 2 stones

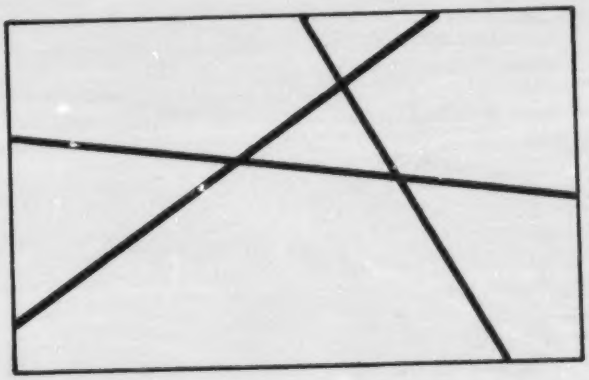
Now use
4 stones.
Note that only
one stone is
allowed in each
space.



answer with
4 stones.

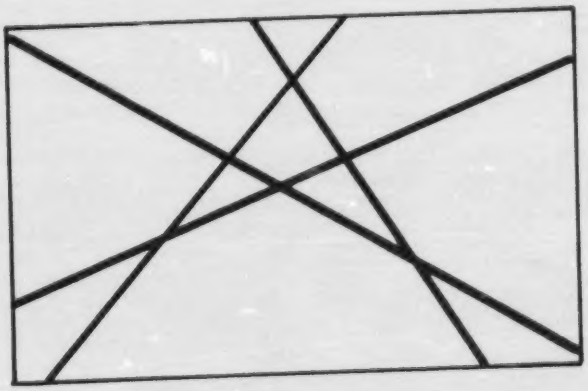
Puzzle 1

Use 2 stones on this diagram.
Then try 4.
Then try 6.
Remember that only one stone is allowed in each space.



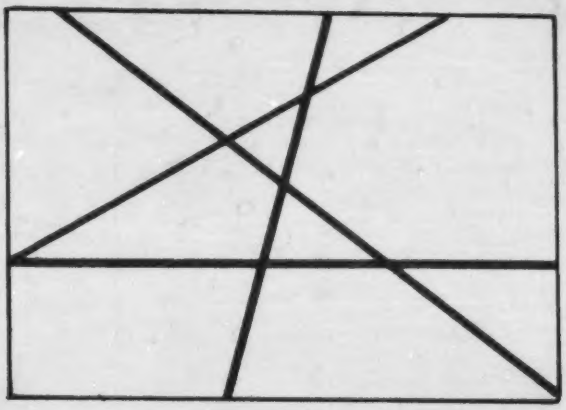
Puzzle 2

Use 2 stones.
Then 4 stones.
Then 6.
Then 8.



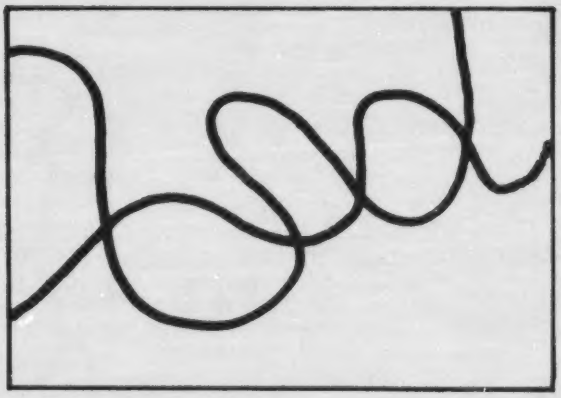
Puzzle 3

Use 2 stones.
Then 4 stones.
Then 6 stones.



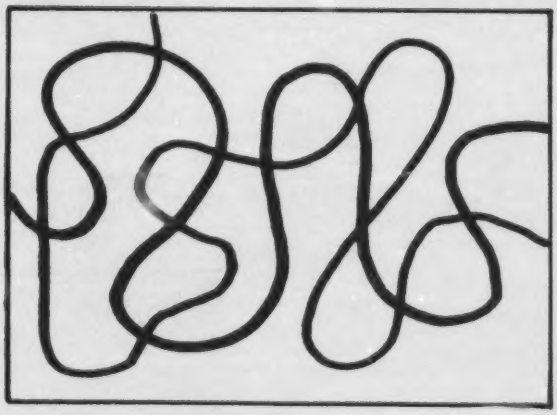
Puzzle 4

Use 6 stones.
There must be
3 on either side
of each line.



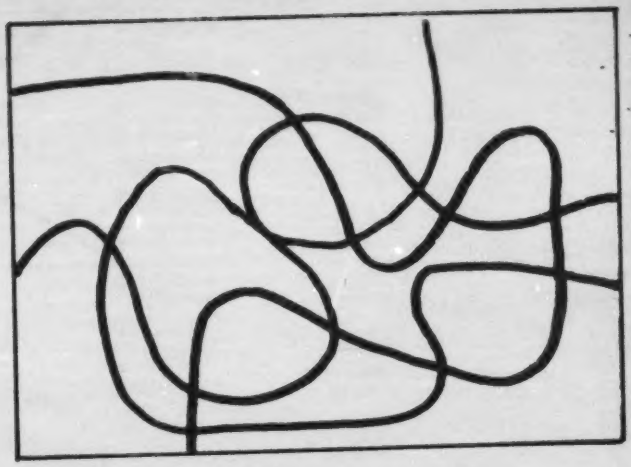
Puzzle 5

Use 14 stones.
There must be
7 on either
side of each line.



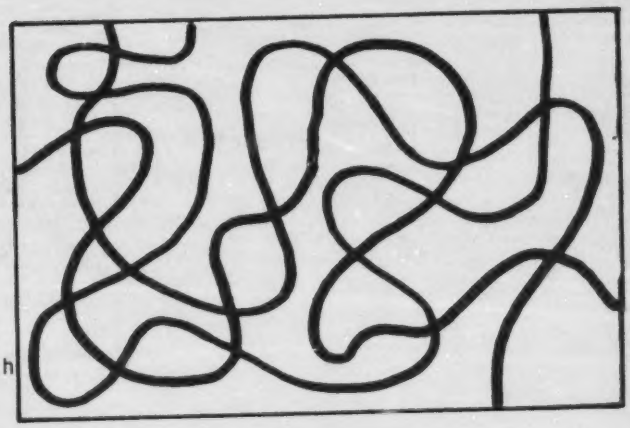
Puzzle 6

Use 12 stones
There must be 6
on either side
of each line.



Puzzle 7

What is the
greatest number
of stones you can
place on this dia-
gram so that there
is an equal number
on each side of each
line?



INVESTIGATION

1. Can you find a system of marking each space in Puzzle 7 so that it is easier to find the largest possible answer?

2. In the two curved line puzzles, like puzzles 4 and 5, can you find a connection between the number of intersections and the most stones than can be placed?

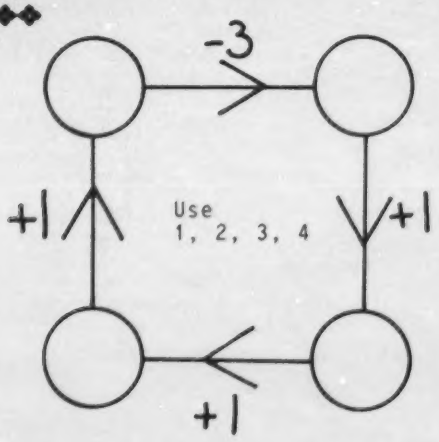
TRAILS

PUZZLE 1

Take stones numbered.

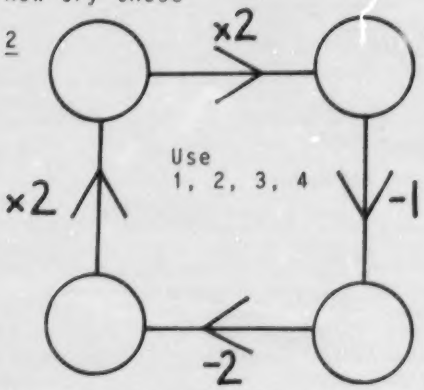
① ② ③ ④

Put one in each circle so that the arrows become true.
For example, if you put 4 in the first circle then 1 must go in the next because 4 - 3 = 1

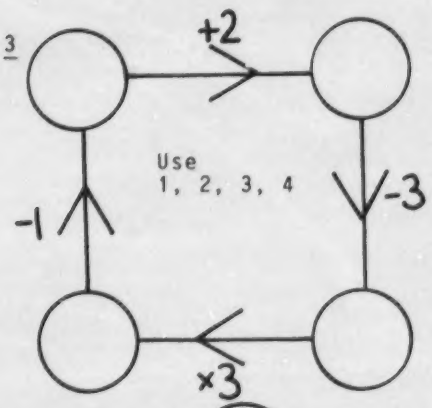


Now try these

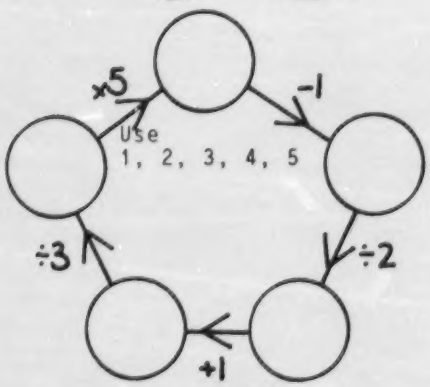
2



3



4



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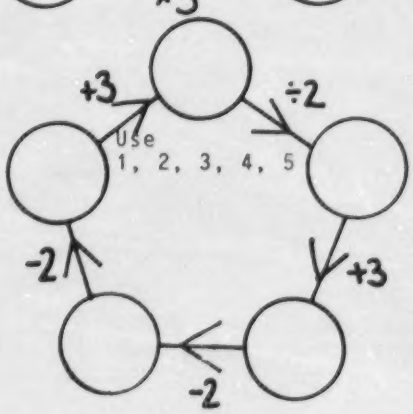
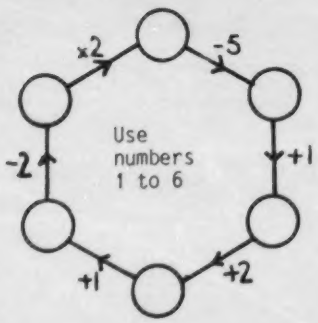
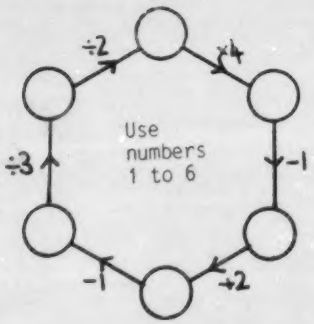


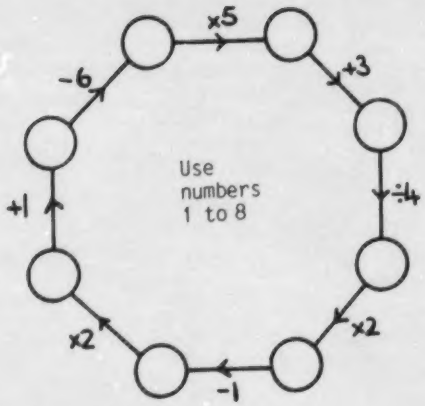
6


×2
-5
-2
+1
+1
+2
Use
numbers
1 to 6

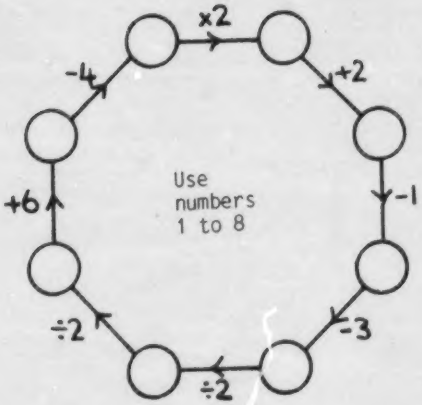
7


÷2
×4
÷3
-1
-1
+2
Use
numbers
1 to 6

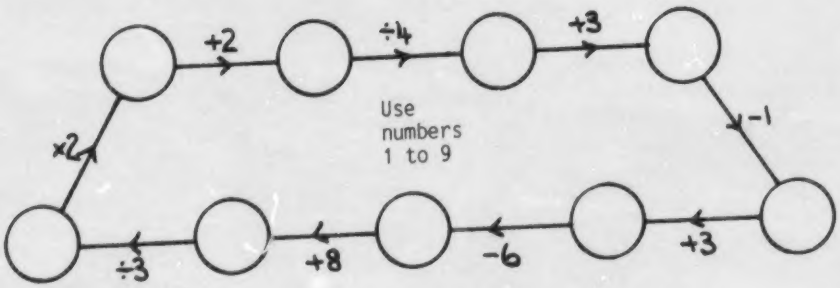
8


×5
-6
+3
+1
÷4
×2
×2
-1
Use
numbers
1 to 8

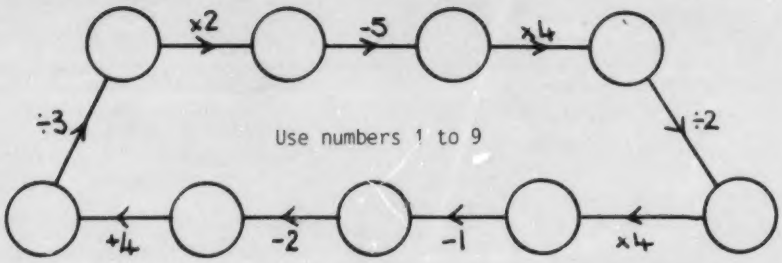
9


×2
-4
+2
+6
-1
÷2
-3
÷2
Use
numbers
1 to 8

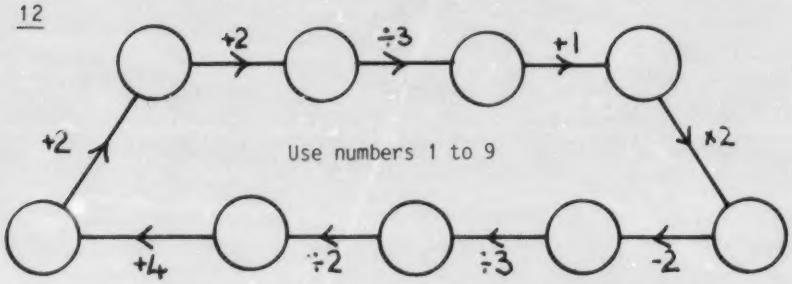
10


+2
÷4
+3
×2
-1
÷3
+8
-6
+3
Use
numbers
1 to 9

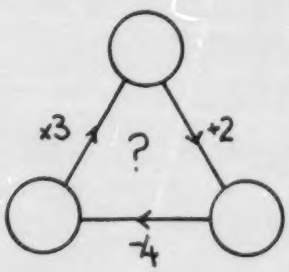
11



12



INVESTIGATION

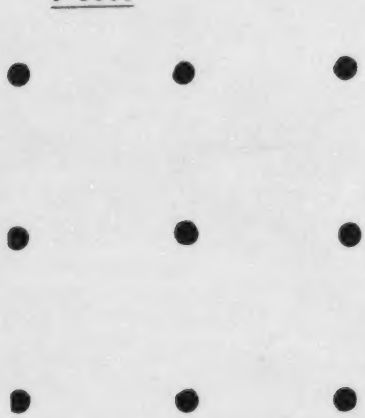


1. In this triangle trial can you work out which numbers go in the circles?

2. Make up similar puzzles and find a method for solving them.
Try and make puzzles which only have one possible solution.

DOTS

9 Dots



These are rows.

These are columns.

Puzzle 1

Take 3 stones. Place them on the dots so that each row has an odd number of stones in it, and each column has an odd number of stones in it. No rows or columns can be empty.

Puzzle 2

Now take 5 stones. Place them so that there is an odd number of stones in each row and each column.

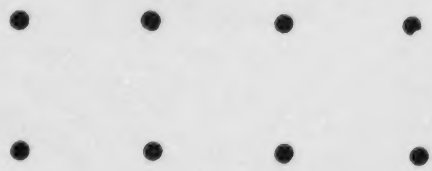
Puzzle 3

Now take 6 stones. Place them so that there is an even number of stones in each row and each column.

16 DOTS

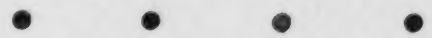
Puzzle 4

On the 16 dots place 4 stones so that there is an odd number of stones in each row and each column. (Remember: no lines must be empty.)



Puzzle 5

Now use 6 stones. Make all rows and columns odd.



Puzzle 6

Use 8 stones. Make all rows and columns even.

Puzzle 7.

Still with 8 stones make all rows and columns odd!

Puzzle 8

Still with 8 stones, make all rows even and all columns odd.

Puzzle 9

Now with 10 stones, make all rows and columns even.

Puzzle 10

Still with 10 stones, make all rows and columns odd.

Puzzle 11

Still with 10 stones, make all rows even and all columns odd.

Puzzle 12

Now with 12 stones, make all rows and columns even.

What other puzzles could you make up with 12 stones on this pattern of dots?

Investigation: 25 Dots

1. Place 5 stones on the dots so all rows and columns are odd. Is it possible to place 7 stones and make all rows and columns odd? What other numbers of stones can be used to make all lines odd?

2. Now try and make all rows and columns even. What numbers of stones can you use?

3. On the 16 dots it was possible to arrange 10 stones to make either odd rows or even rows. Can you find a number of stones on 25 dots which can be arranged in this way? Will there be a solution for 36 dots?

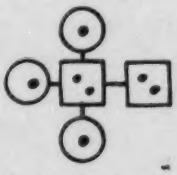
SIMON'S PUZZLE

In these puzzles there are two rules:

(1) you must put an equal number of stones in all shapes which are the same.

(2) you must make the total number of stones in a line equal the number shown by the arrow.

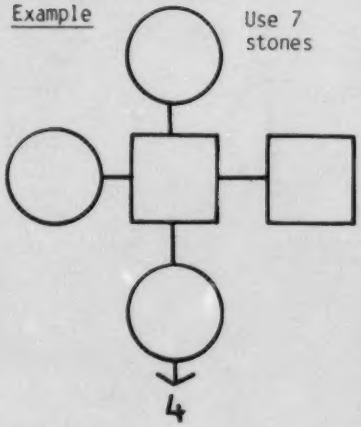
Answer:



Each circle has 1 stone.
Each square has 2 stones.

Example

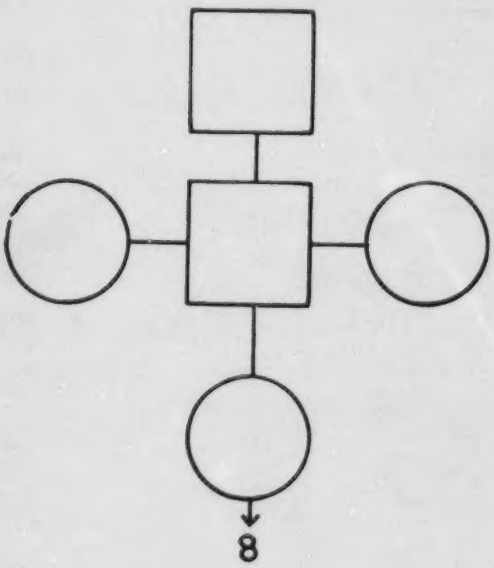
Use 7 stones



Now try try these.

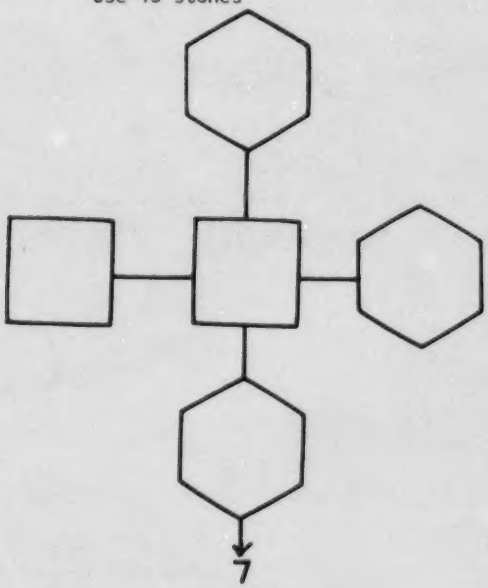
Puzzle 1.

Use 12 stones



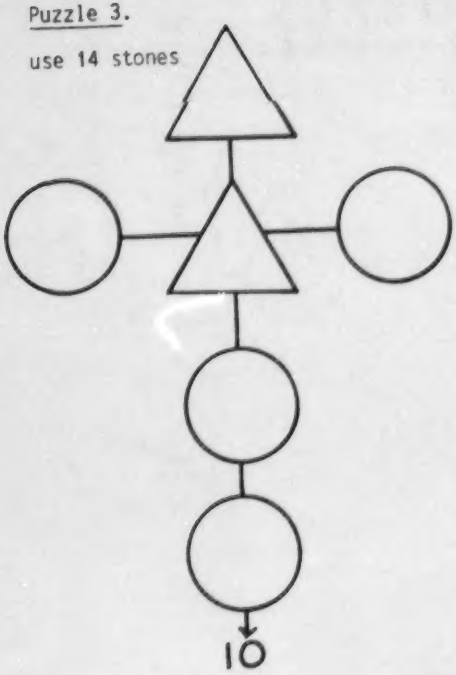
Puzzle 2

Use 13 stones



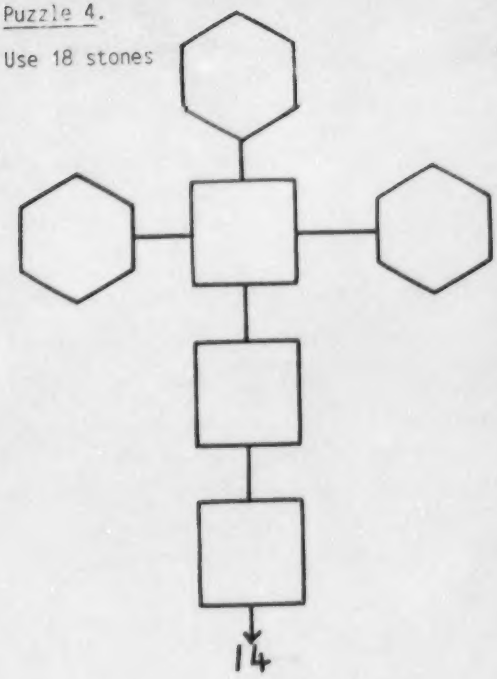
Puzzle 3.

use 14 stones



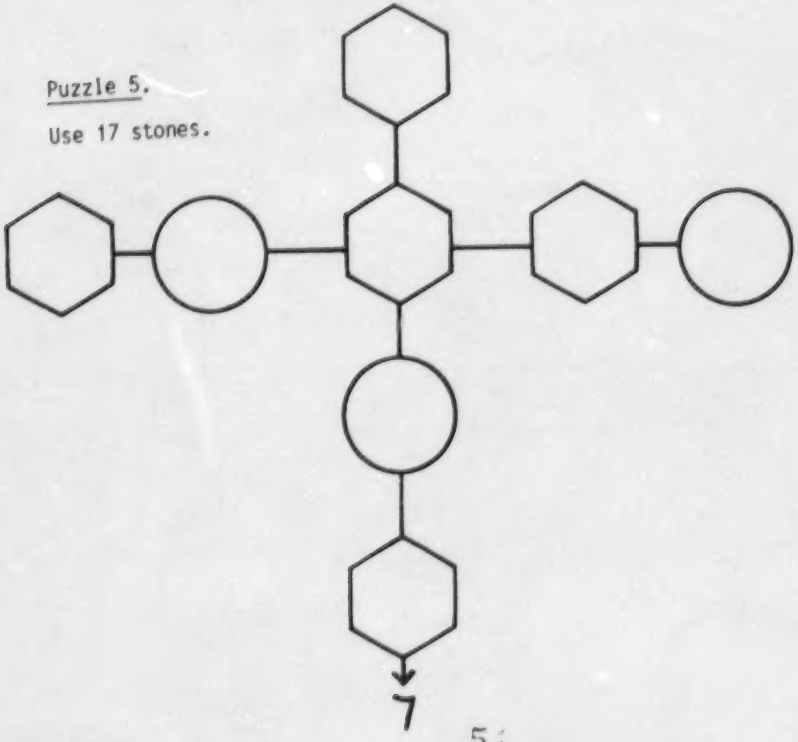
Puzzle 4.

Use 18 stones



Puzzle 5.

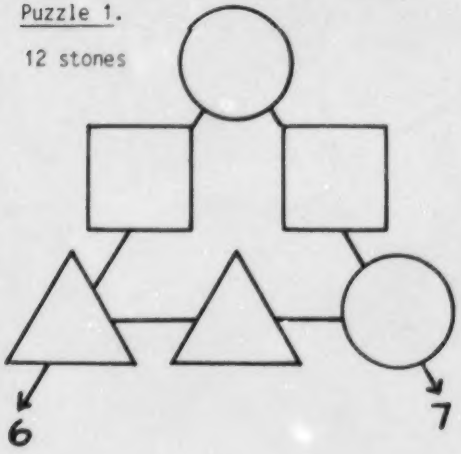
Use 17 stones.



Now try these with 3 different shapes.

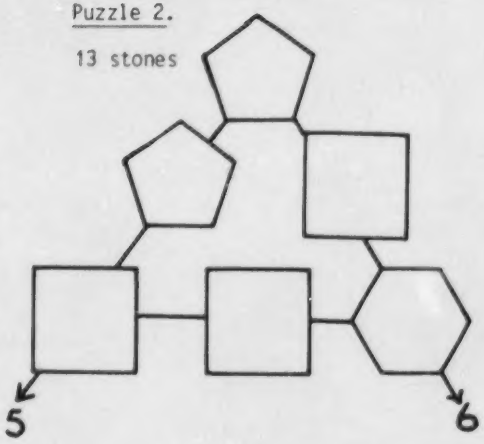
Puzzle 1.

12 stones



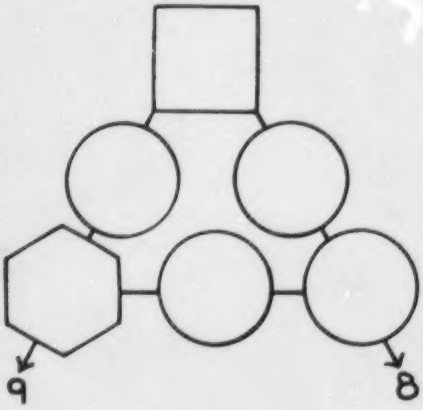
Puzzle 2.

13 stones



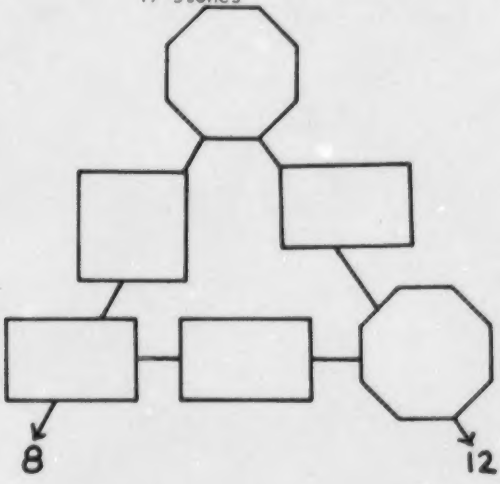
Puzzle 3.

18 stones



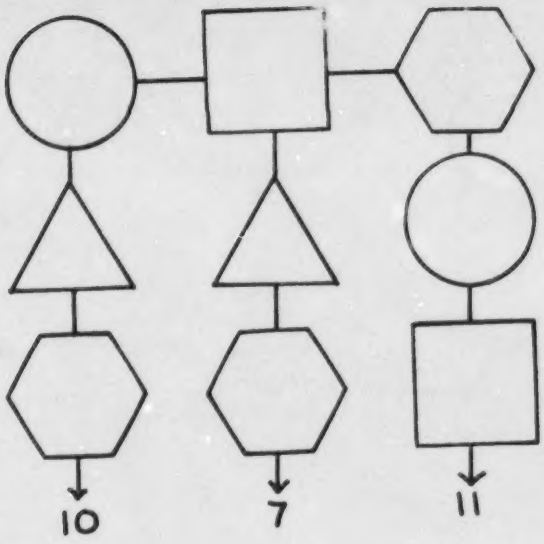
Puzzle 4.

17 stones

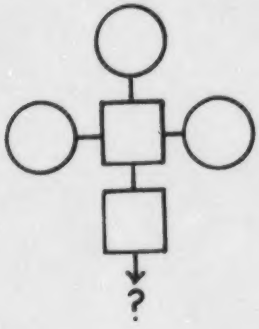


Puzzle 5.

Use 28 stones



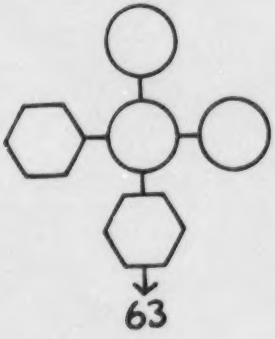
Investigation.



(1) In Puzzle 1 this diagram was filled using 7 stones. Can you find a solution with 8 stones? What will be the number at the end of the arrow?

(2) What other numbers of stones can be used on this puzzle? Are there some which are impossible? (You can have empty shapes if you like.)

(3) Can you work out an algebraic method for solving all these puzzles?
Use your method to solve this puzzle which needs 100 stones.



PILE UP

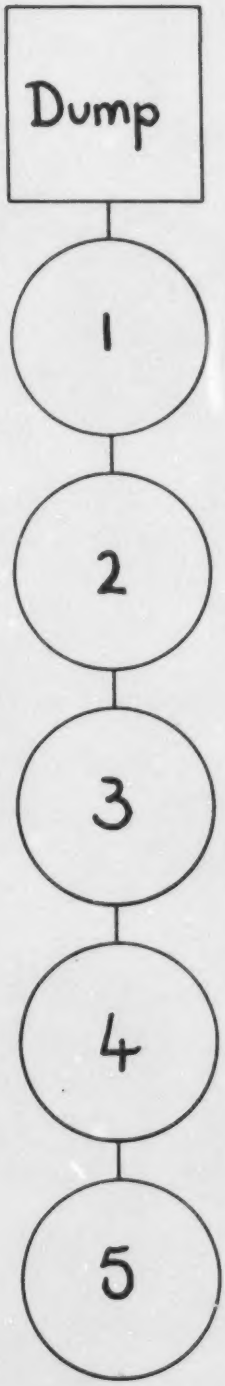
The aim of these puzzles is to 'pile up' all the stones in the large square, the 'dump'. The rules are as follows:-

(1) If there is 1 stone in a circle it may move one step towards the dump:
eg.

If there are 2 stones they move 2 steps, if 3 stones then 3 steps. The number of steps is equal to the number of stones.

(2) All the stones in a circle move together. All moves must be towards the dump.

(3) Stones must land exactly in the dump and not overshoot.



PUZZLE 1

Put 1 stone in each of the circles 1,2 and 3. Can you pile them up in the dump in just 3 moves?

Puzzle 2

Start with single stones in circles 1,2,3 and 4. Can you pile them up in 7 moves?

Puzzle 3

Start with single stones at 1,2,3,4 and 5. Can you pile them up in just 6 moves?

Puzzle 4

Start with single stones in circles 1 to 6. Can you pile them up in just 7 moves?

Puzzle 5

Start with single stones in circles 1 to 7. Can you pile them up in just 7 moves?

Puzzle 6

Try 8 stones, one in each circle 1 to 8. Can you pile them up in just 11 moves?

Puzzle 7

Now start with 9 stones and pile them up in 12 moves.

Puzzle 8

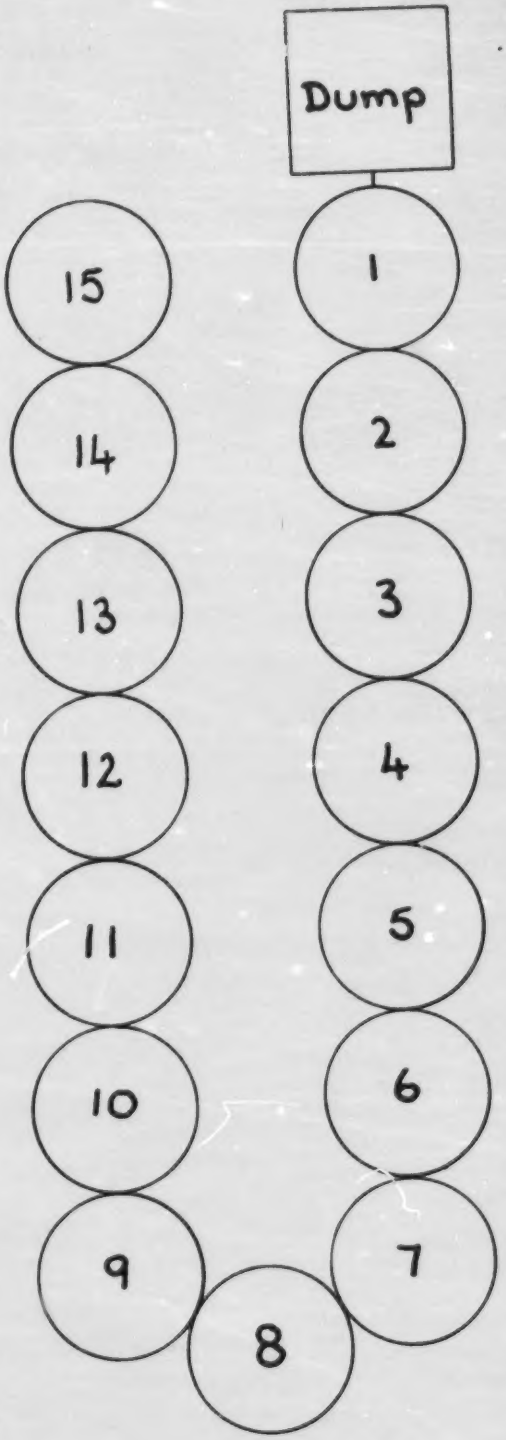
Can you pile up 10 stones in just 13 moves?

Puzzle 9

Can you pile up 11 stones in 12 moves?

Puzzle 10

Can you pile up 15 stones in 15 moves?



INVESTIGATION

(1) In which of the puzzles do the number of moves equal the number of stones?

Can you predict which will be the next puzzle where this will be true?
What series do these numbers belong to?

(2) If the rules are changed slightly we get a new puzzle. The new rule is that the stones may move either towards or away from the dump.

Now try the puzzles again using this rule.

Can you find a connection between the number of stones and the number of moves?

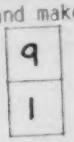
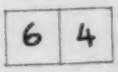
(3) Can you develop a method of moving the stones, so that no matter how many you start with you can be sure of piling them up in the fewest possible moves.

JUST PUZZLES

COVER UP

Puzzle 1

(a) Take two stones and cover a pair of numbers which are next to each other and make ten.

eg.  or 

but not

Try and cover 8 pairs altogether.

(b) Clear off the stones and cover 6 pairs that make 11.

(c) Clear off the stones and cover 7 pairs that make 12.

9	3	7	3	9
4	7	4	8	2
8	5	6	4	11
7	8	5	6	1
6	4	9	1	9

(d) Clear off the stones and cover 6 pairs that make 13.

Puzzle 2

This time first cover a pair that adds to 1, then cover a pair that adds to 2. Continue in this way covering 3, then 4 up to 12. One number is left uncovered; which is it?

Sometimes there are several ways to make the number. You have to choose the pair that makes all other pairs possible.

3	6	6	1	3
6	2	3	4	7
1	3	1	1	3
0	3	5	1	2
4	2	5	5	6

Puzzle 3.

10	5	11	8	2	4	5
12	1	7	14	5	3	0
11	0	4	2	7	9	8
9	5	8	8	7	2	2
9	1	17	14	1	1	7
5	7	5	10	8	4	3
6	3	2	11	9	7	9

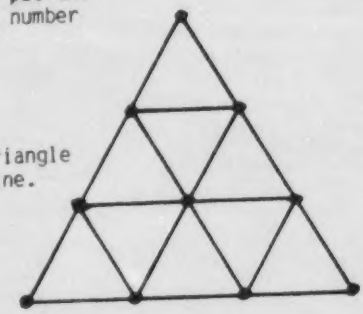
First cover a pair that adds to 1.
Then cover a pair that adds to 2. Continue in this way, next covering a pair that makes 3, then 4 up to the last pair which make 24.

One square will be left uncovered.
Which is it?

HEXAPLA

Puzzle 1:

Use the triangle. Take 6 stones and put them on the dots so that there is an even number on each line.

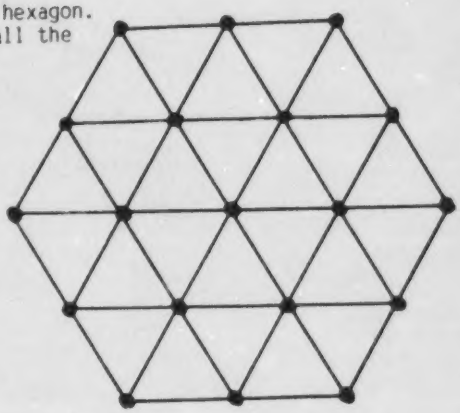


Puzzle 2:

Now rearrange the 6 stones on the triangle so there is an odd number on each line.

Puzzle 3:

For the rest of the puzzles use the hexagon. Place 5 stones on the dots so that all the lines have an odd number of stones.



Puzzle 4

Use 7 stones. Make all lines odd.

Puzzle 5

Use 9 stones. Make all lines odd.

Puzzle 6

Use 10 stones. Make all lines even.

Puzzle 7

Use 11 stones. Make all lines odd.

Puzzle 8

Use 12 stones. Make all lines even.

Puzzle 9

Use 15 stones. Make all lines odd.

Puzzle 10

Use 16 stones. Make all lines even.

SQUARES

4 stones placed like this:

Make 1 square; 6 stones like this:

Make two squares.

Diagram 1.

Puzzle 1.

Place 7 stones so that they make 3 squares.

Puzzle 2.

Place 8 stones so that they make 4 squares.

Puzzle 3.

Place 9 stones so that they make 6 squares.

Now try these harder puzzles.

Puzzle 4.

Can you put 6 stones on the nine dots so that not one square is formed?

Puzzle 5.

How many can you put on the 16 dots (use diagram 1) so no square is formed?

Puzzle 6.

How many can you put on the 25 dots so no square is formed?

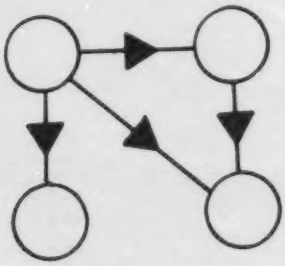
FACTORS

You will need stones marked 1 to 10
in all the diagrams the arrow → means
'is a factor of'. For example

2 is a factor of 4.

Puzzle 1.

Use stones 1, 2, 3, 4. Place them so that the diagram is correct.

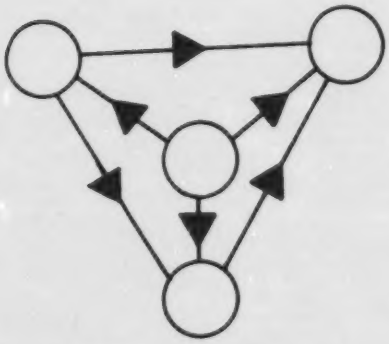


Puzzle 2.

On the same diagram use stones 2, 4, 6, 8.

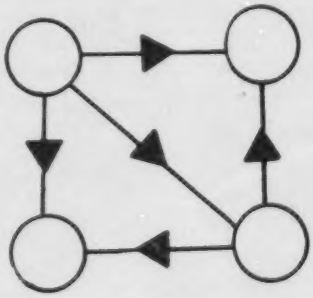
Puzzle 3.

Use stones marked 1, 2, 4, 8.



Puzzle 4.

On this diagram use stones 1, 3, 6, 9.



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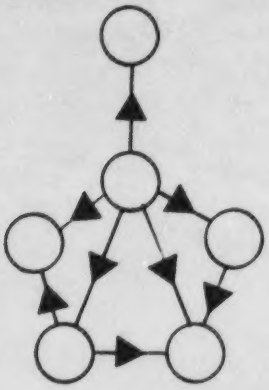
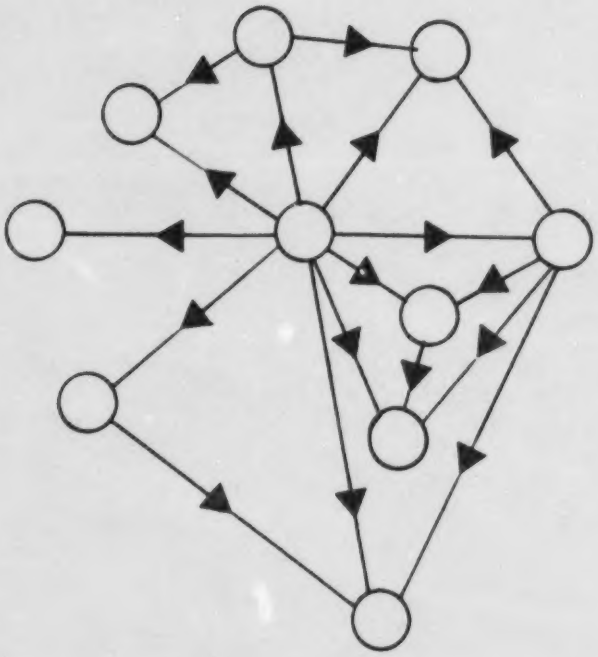
Puzzle 5.



Use stones marked
1, 2, 3, 4, 5, 6.

Puzzle 6.

Use stones marked
1, 2, 3, 4, 5, 6, 7,
8, 9, 10.



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LINES

5 stones - place the 5 stones on the dots so that they make 2 straight lines with 3 stones in each line.

6 stones - place the 6 stones to make 3 lines with 3 stones in each line.

7 stones - place the 7 stones to make 6 lines with 3 stones in each line.

9 stones

Place the 9 stones to form 9 lines with 3 stones in each.

9 stones

Place the 9 stones to form 10 lines with 3 on each.

A bit of a trick : Arrange 6 stones so that they form two lines with 4 stones in each line.

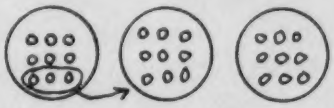
THIRDS

These are the rules:-

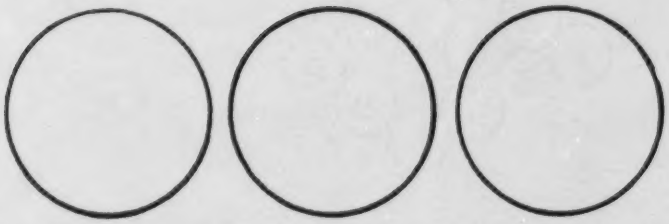
(1) Start with 9 stones in each circle.

(2) On each move you are allowed to move ⅓ of the stones in a circle to another circle:

Example:



2 mc e Puzzles



In just 2 moves re-arrange the stones so that there are distributed

(1) 6,6,15 (2) 11,4,12

(3) 8,13,6 (4) 8,9,10

(5) 14,9,4

3 move Puzzles

In just 3 moves re-arrange the stones into groups of

(1) 17,6,4 (2) 15,8,4

(3) 6,10,11 (4) 14,7,6

(5) 4,10,13

There is one other new arrangement that can be made in 3 moves. Can you find it?

4 Move Puzzles

In just 4 moves re-arrange the stones into groups of

(1) 14,4,9 (2) 16,7,4

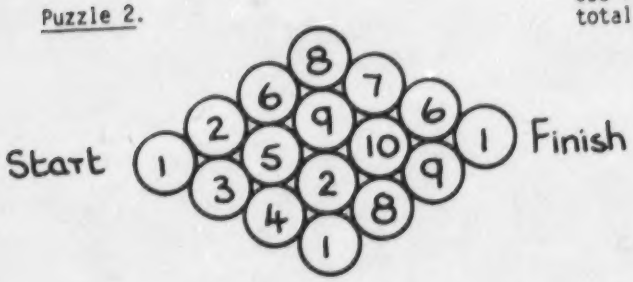
STONE PATHS

Puzzle 1.

Make a path across the puzzle with stones. Place the stones so that the numbers covered add to 17.

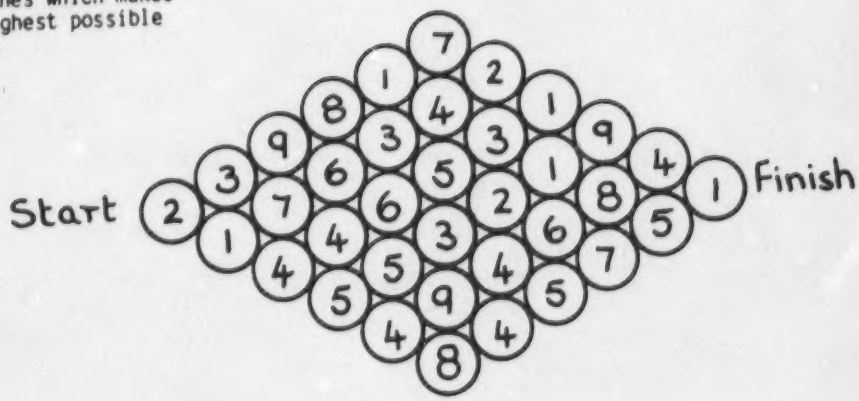
Puzzle 2.

Use 7 stones and make a total of 35.



Puzzle 3.

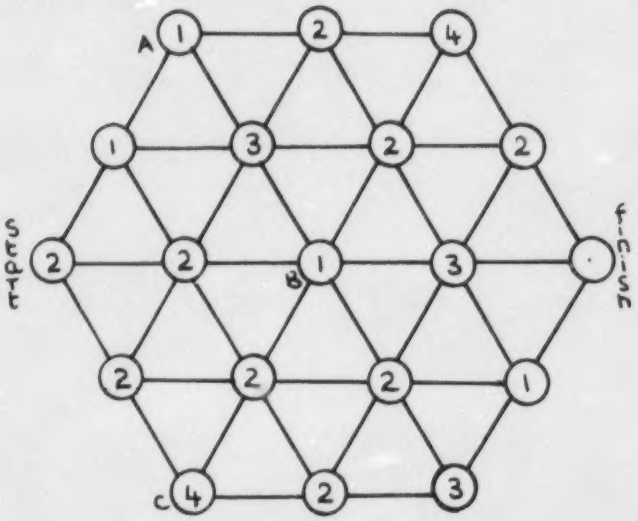
Make the path with 11 stones which makes the highest possible total.



MAZES

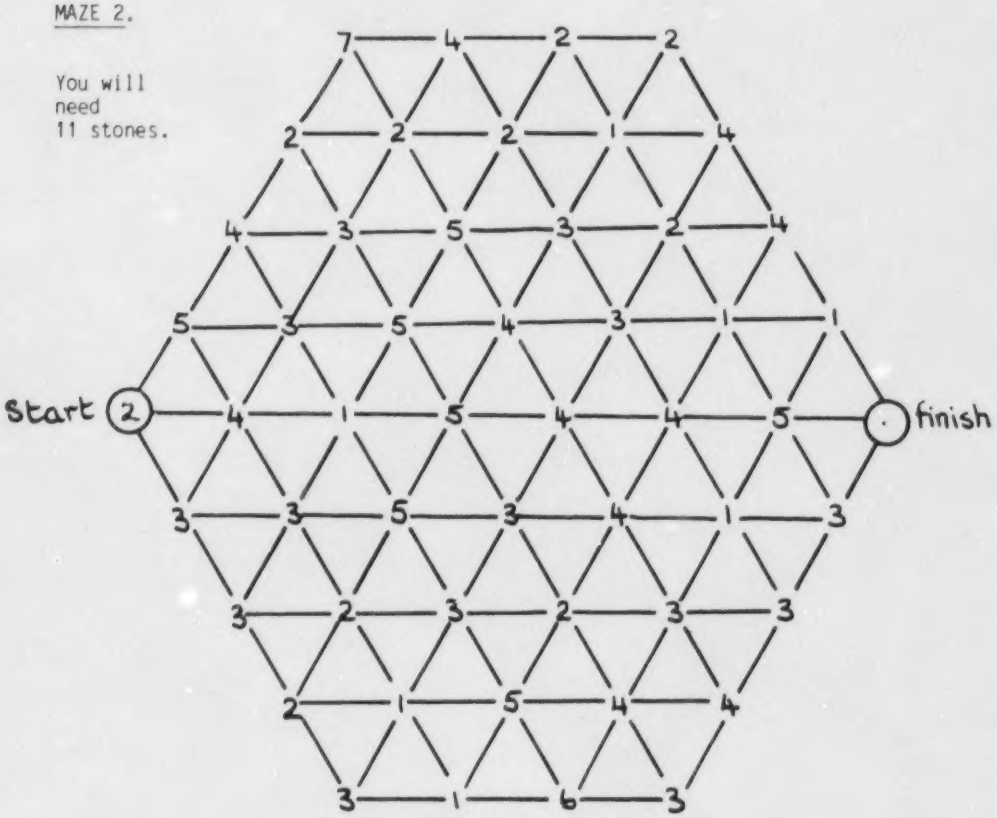
MAZE 1

Put a stone at the start. Here you will see a '2'. This means you can place your next stone 2 steps away in any direction? at A,B or C. Choose which you want to cover and look at the number. This tells you how many steps you must take to your next stone. Try and get a stone on the finish! You will need 6 stones altogether.



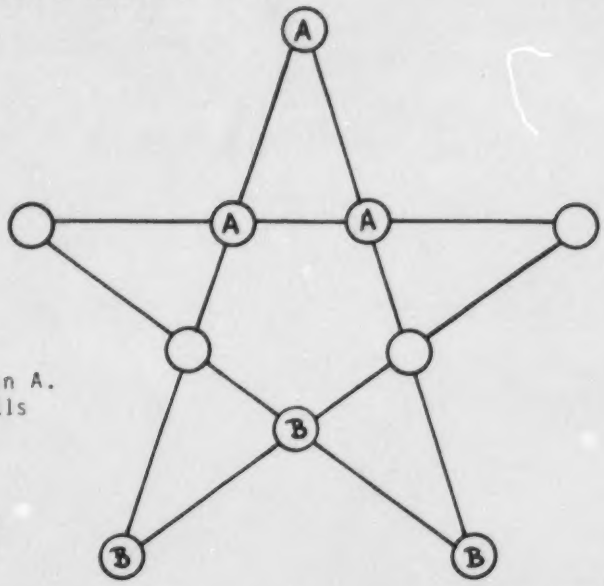
MAZE 2.

You will need 11 stones.



GAMES

Star game - for 2 players.



One player puts 3 stones on the circles marked with an A. The other player puts 3 shells on the circles marked B.

The player with the stones goes first. The aim of the game is to capture your opponent's pieces.

Moves are made by sliding one of your pieces along a line to an empty circle.
Captures are made by jumping over your opponent's pieces into an empty circle. You can jump over one, two or even three pieces in one go.
When a piece can jump it must do so.

You will find that in order to win you may have to 'sacrifice' one of your pieces. This means moving it so that the other player has to take it but then you may be able to take two or even three of his pieces.

By the way, this board can be drawn in one movement.

GRAB

A game for any number of players which gives practice in division. Place a large number of stones in a pile. Each player takes a handful of stones from the pile and then counts to see how many he has got.
If the number taken is divisible by 2, score 2 points.
If the number taken is divisible by 3, 3 points, if by 5 then 5 points.

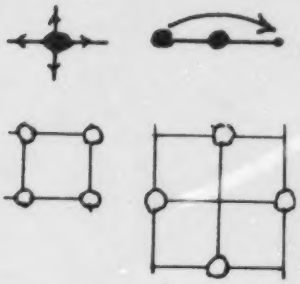
For example: if you take 12 score 2 +3 = 5
: if you take 15 score 3 +5 = 8

The first player to score 50 is the winner.

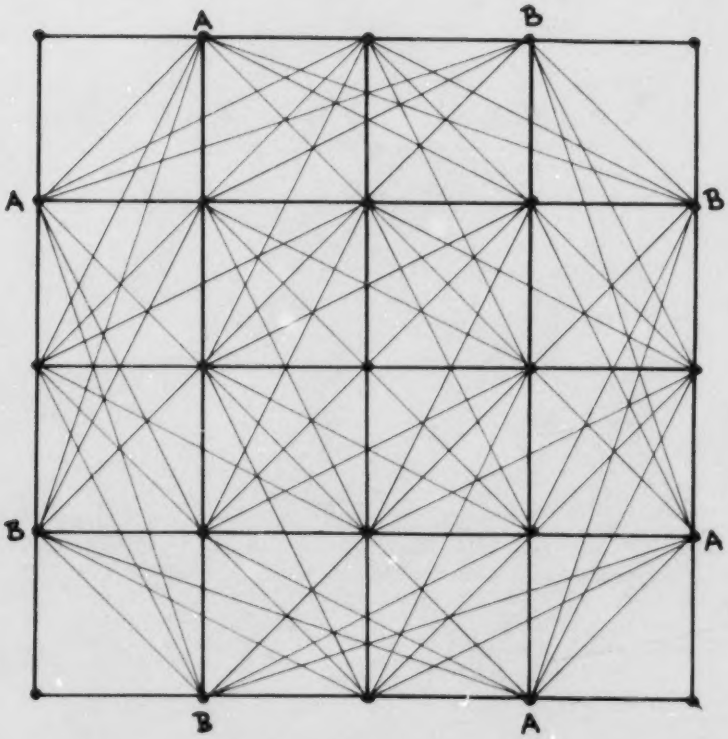
QUADRANGLE

A Game for 2 players:

One player has four shells the other has 4 stones.



1. Start with the shells at points MARKED A, and with stones at points MARKED B.
2. Each player moves a piece in turn, one step along a line to an empty square or jumps over another piece to an empty space.
3. The winner is the first player to get his 4 pieces on the corners of a square.



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Stone Bridge : A game for 2 players.

Each player starts with 10 stones each. Then each player in turn chooses numbers from 1 to 20 and writes them on a piece of paper. Continue choosing till all numbers have been chosen. Then play in this way. Each player hides his stones behind his hands. Then he hides a certain number in his right hand and holds them out. Both children then open their right hands and find the total of stones. The child who has this number written on his paper wins all the stones. Continue in this way till one child has won all the stones.

An example of a game.

Mary chooses 1,2,4,5,6,9,12,13,16,20

Peter chooses 3,7,8,10,11,14,15,17,18,19.

First go: Mary puts out 6 in her right hand.
Peter puts out 5.
Total is 11. Peter has 11 in his list so he wins all 11 stones and adds them to his pile.

This game can be played at a very simple level just to practice addition and number recognition. But there is also a good deal of skill in the choosing of the numbers and then in deciding how many stones to put out. You can if you wish start with only 5 stones each and choose numbers from 1 to 10 or increase the number of stones and the numbers chosen.

HIDE AND SEEK

A game for 2 players which gives practice in addition and subtraction. Place 20 stones between you. Then each take some stones from the pile and hide them behind your hand so that you can see them but the other player cannot. Now work out how many stones the other player has got hidden. If you guess correctly you score 1 point. If he guesses your number he scores 1 point also. Put back the stones and start again. The first player to score 10 points wins.

You can start with fewer or more stones in the pile depending on the ability of the players.

SHELL OUT

A Game for 2 players

Each player has 10 stones. The first player to get rid of all his stones is the winner.

How to play: The first player chooses any circle and puts the number of stones shown in that circle.

The second player then can choose either of the two circles next to the one chosen by the first player. He fills it with his stones.

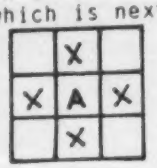
Each player continues in turn choosing one of the circles next to those already chosen. The first player to get rid of all his 10 stones is the winner but he must go out exactly. If you have 3 stones left you cannot place them on a 4 circle.

Try playing the game with 12 stones.
Try making up a board of your own.

BOXES. 2 Players.

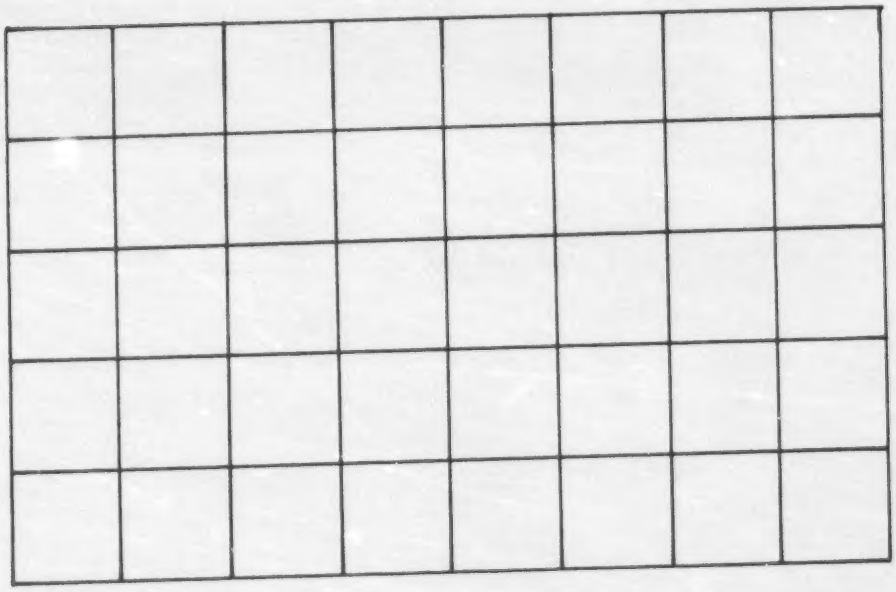
Each player takes it in turn to place a stone in a box. The stone must not go in a box which is next to one with a stone in

Example.



If a stone is placed in box A then the next player can not put stones in the boxes marked X.

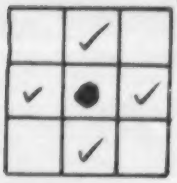
The player who cannot go loses.



DEAD END 2 Players

Each player takes it in turn to place a stone in a square. The first player may put his first stone wherever he likes. Players now take it in turn to place a stone in a square next to the last one played.

Example

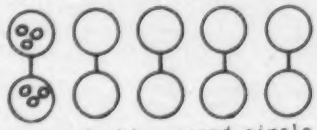


If the first player put a stone as shown then the next player could put a stone in any of the boxes shown with a tick. The player who cannot go loses.

MATCHING GAME.

For 2 players. Each player has 12 stones to start with.
The first player puts one, two, three, four or five stones into his first circle. The second player must match in his circle whatever the first player puts.

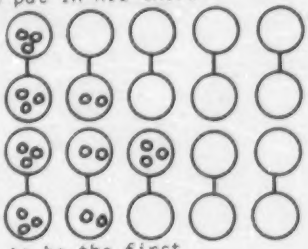
Example. Player 1 puts 3 stones
Player 2 must put 3 stones



Now the second player chooses how many stones to put in his second circle. Player 1 must now match this and then choose what to put in his third circle.

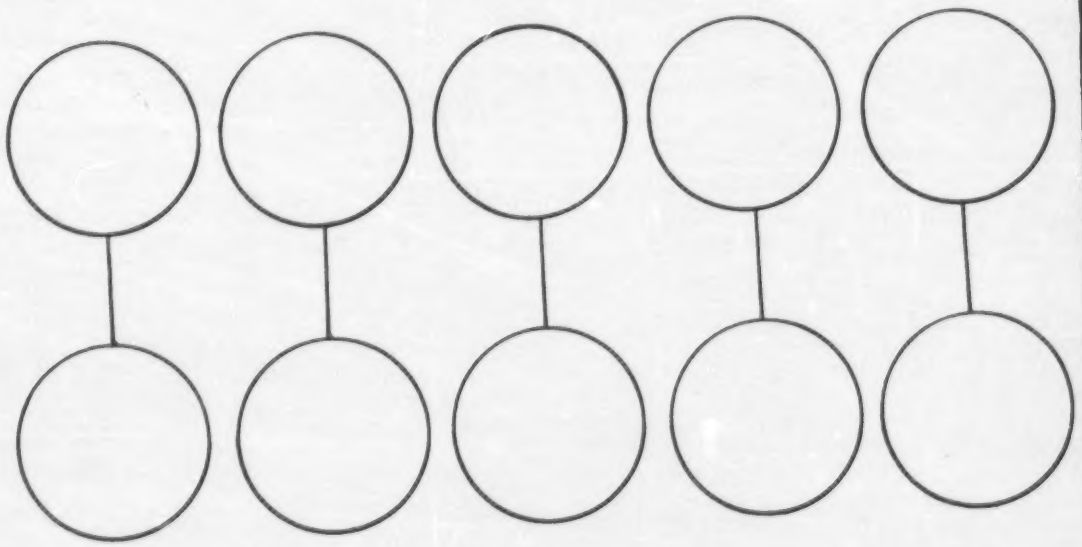
Example. Player 2 decides to put 2 stones in the second circle.

Now Player 1 matches this and chooses to play 3 stones in the third circle.



Play continues in this way. The aim of the game is to be the first player to get rid of all 12 stones.

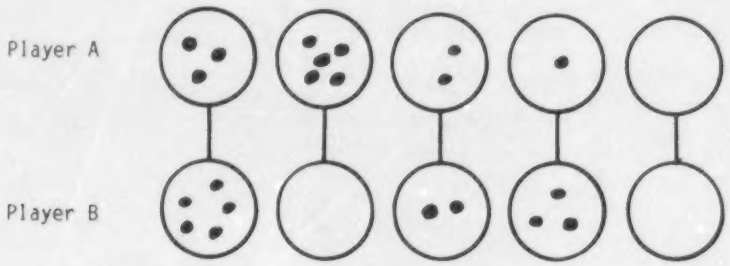
Board for Matching Game.



CAPTURE.

A game for two players. Use the same board as in the matching game, but this time each player has 10 stones. Put a book upright between the circles so that each player's side is hidden from the other. Each player places his stones in any way he likes in his 5 circles, but he may not place more than 5 stones in any circle. When each player has put out his pieces the book is removed. Now compare the number of stones in linked circles. The higher number captures the lower number except where 5 is opposite 0. Then 0 captures the 5.

Example



In the first pair : 5 captures 2.
In the second pair : 0 captures 5.
In the third pair : This is a draw. No one is captured.
In the fourth pair : 3 captures 1
In the fifth pair : a draw.

The captured stones are added to the stones already held. Continue the game until one person has captured all the stones. This person is the winner.

SCATTER GAMES.

These games are all played by dropping stones from a certain height onto a flat surface.

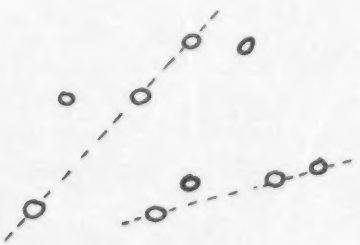
If you are dropping onto a hard surface, a table, or concrete floor then hold the stones in your right hand and place it on top of your left hand. This measures the distance you should drop from. Remove your left hand and open your right hand. The stones should scatter but not go too far.

If you are playing on a soft surface, sand, or earth, for example then you need to drop the stones from higher. Put your left elbow on the ground and now place your right hand on your left. This measures the height.

Game 1.

Drop 9 stones. For every three stones that lie in a straight line count three points. Use a coconut rib to check which are in a line. If four are in a line count four. First player to 30 wins.

Example.



Score: 6 points.

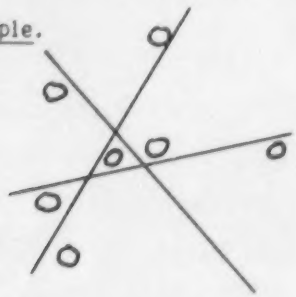
Game 2.

Each player chooses a small shell as his own. All the shells plus a stone are held in the hand and dropped. The player whose shell is closest to the stone wins the stone. The first player to collect 10 stones wins.

Game 3.

Drop 7 stones. If the other player can place 3 coconut ribs so that all seven stones have a 'field' of their own he scores a point. Take it in turns to drop the stones. First to 10 points wins.

Example.

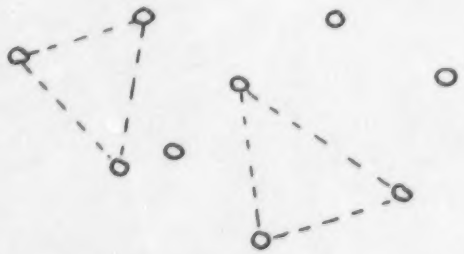


Each stone is in its own field.

Game 4.

Drop 9 stones. Any 3 stones that make an acute angled triangle can be removed. Use a piece of paper to check that the angles are less than 90^0.

Example



Score 1 point for each triangle you can remove. First to 20 points wins.

ANSWERS AND NOTES

Routes - answers and notes.

Page 1

(1) 5 moves

A - 2 - B
A - 2 - B
A - 3 - B
A - 2 - B
A - 2 - B

(2) 6 moves

A - 2 - B
A - 2 - B
A - 3 - B
A - 2 - B
A - 2 - B
A - 2 - B

(3) 5 moves

B - 3 - A
B - 2 - A
B - 3 - A
B - 2 - A
B - 3 - A

(4) 4 moves

B - 3 - A
B - 3 - A
B - 2 - A
B - 3 - A

Page 2

(5)

A - 5 - C
A - 4 - C
A - 5 - C
A - 4 - C
A - 5 - C

(6)

C - 4 - A
C - 3 - B
A - 3 - B
B - 2 - A
B - 2 - A
B - 2 - A

(7)

C - 4 - A
C - 4 - A
A - 3 - B
B - 2 - C
C - 3 - A
B - 2 - A
B - 2 - A

(8)

A - 5 - C
A - 3 - B
C - 3 - B
B - 2 - C
B - 2 - C
B - 2 - C

Puzzle 9

A-3-B, B-2-A, A-3-B, B-2-C, B-2-C, A-5-C.

Puzzle 10

Page 3

(1)

A - 2 - B
A - 3 - C
C - 2 - B
B - 3 - D
D - 3 - C
C - 4 - D
D - 3 - C
C - 2 - B
B - 3 - D
D - 3 - C
C - 4 - D

(2)

A - 2 - B
A - 2 - B
A - 2 - B
B - 3 - D
B - 3 - D

(3)

A - 2 - B
A - 2 - B
A - 3 - C
C - 2 - B
B - 3 - D
B - 3 - D
D - 3 - C
C - 4 - D

(4)

A - 3 - C
A - 3 - C
A - 3 - C
C - 4 - D
C - 2 - B
B - 3 - D
D - 3 - C
C - 2 - B
B - 3 - D
D - 3 - C
C - 4 - D

(5)

A - 3 - C
A - 3 - C
A - 3 - C
C - 4 - D
C - 4 - D
D - 3 - C
C - 4 - D

10 stones - 8 moves, 11 stones - 11 moves,
12 stones - 7 moves.

Harder Puzzle - answers

Page 4

(1) 5 moves: A - 3 - B
B - 2 - E
E - 2 - A
A - 3 - B
B - 4 - C

(2) 5 stones take 6 moves: A - 3 - E
E - 2 - A
A - 3 - E
E - 2 - A
A - 3 - E
E - 5 - C

6 stones take 5 moves: A - 2 - D
A - 2 - D
A - 2 - D
D - 3 - C
D - 3 - C

7 stones take 14 moves: A - 2 - D
A - 2 - D
A - 2 - D
D - 3 - E
E - 2 - A
A - 3 - E
E - 2 - A

cont. E - 2 - A
A - 3 - B
B - 2 - E
E - 2 - A
A - 3 - E
B - 4 - C
D - 3 - C

8 stones is now clearly possible; it could be moved as two sets of 4.
In the same way all other numbers can be made up of combinations of 4,5,6 and 7. The only impossible numbers are 1,2 and 3.

Notes on Investigation

Page 4

Stones	Moves
4	2
5	5
6	3
7	6
8	4
9	7
10	5
11	8

(1) The numbers in the series 2,5,3,6,4,7,5,8 go up by 3 then down 2 repeatedly. Whenever the number of stones is even, 2N, the number of moves is N. If the number of stones is odd, 2N+1, then the number of moves is N+3.
40 stones take 20 moves.
41 stones take 23 moves.

Stones	Moves
3	1
4	3
5	5
6	2
7	4
8	6
9	3
10	5
11	7

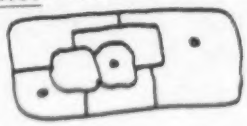
(2) The numbers in the series 1,3,5,2,4,6,3,5,7 go up 2, up 2, down 3 repeatedly. Whenever the number of stones is a multiple of three, 3N, then the number of moves is N. If the number of stones leaves a remainder of 1 when divided by 3, ie. 3N+1, then the number of moves is N+2. If the number of stones leaves a remainder of 2, ie. 3N+2, then the number of moves is N+4.

60 stones take 20 moves.
61 stones take 22 moves.
62 stones take 24 moves.

Page 5

Space Puzzles - answers and notes

Puzzles 1



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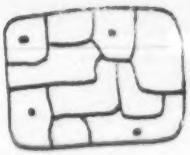

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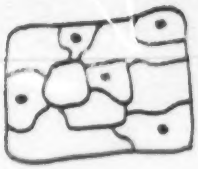
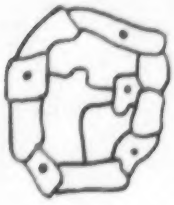


Page 6

4



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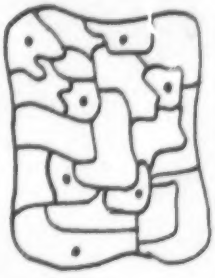

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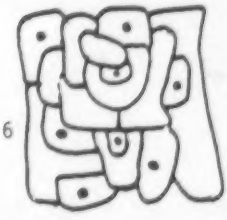

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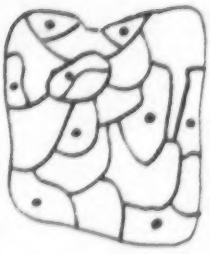

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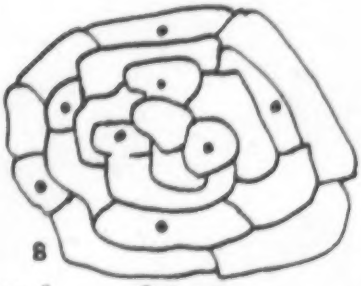
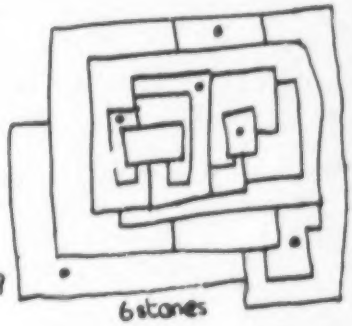


Page 7

8 stones

9



6 stones

Page 9

Investigation

1. To win, put a stone on E. This leaves two spaces C and G still free.
2. If you were first player, play first on H or I.
3. If you were first player, play first on H or I.
4. The two games have exactly the same structure.

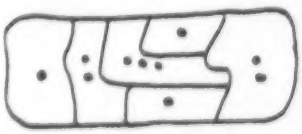
In the Network game the spaces have shrunk to circles and the idea of 'touching' is replaced by 'joined by a line'. For example, space A touches spaces B, I and H. Circle A is joined to circles B, I and H. Identical tactics will be used in the two games.

Page 10

Harder Space Puzzles

1

2



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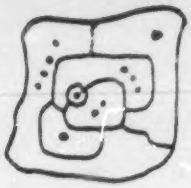

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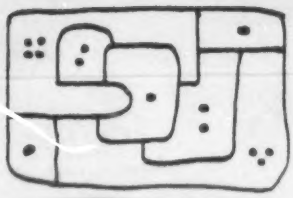

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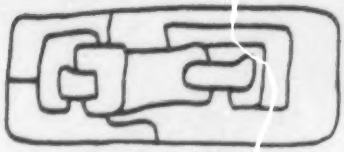

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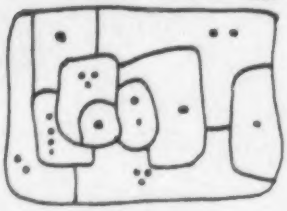
6



7

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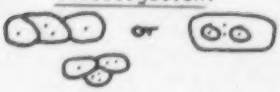
Investigation:



(1) If we assume the spaces must be joined then the least stones for 3 spaces is 4. The most stones needed is 6(1 + 2 + 3).

(2) With 4 spaces the least number of stones is 5. The most number of stones is 10(1 + 2 + 3 + 4).

(3) This investigation is closely related to the 4 colour map problem. This asks us to find the fewest number of colours needed to colour any map so that no two touching spaces are the same colour. Only 4 colours are needed. In our problem with 5 spaces it is impossible to draw them so all 5 touch. You will always find two are not touching. So the greatest number of stones needed is 1 + 2 + 3 + 4 + 1 = 11. As long as we take care to put one stone in each of the non touching spaces the total need never be greater than 11. In the same way it will always be possible to fill 6 spaces with 1 + 2 + 3 + 4 + 1 + 2 = 13 stones. In general if P is the number of spaces the number of stones, S, can be found by dividing P by 4. If $\frac{P}{4}$ = X remainder Y then S = 10X + 1 if Y is 1
S = 10X + 3 if Y is 2
S = 10X + 6 if Y is 3

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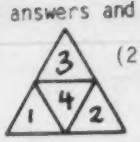
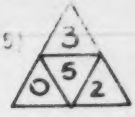
TETRARCHS - answers and notes

Puzzle (1) 

(2)

(3)

(4)

(5) 

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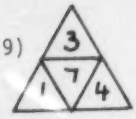
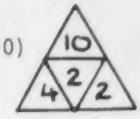
(6)

(7)

(8)

(9) 

(10) 

(11)

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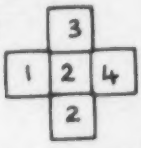
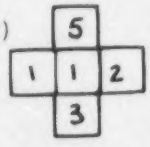
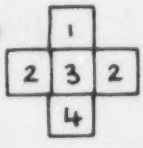
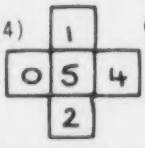
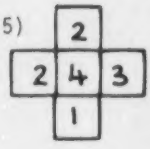
Crosses (1) 

(2) 

(3) 

(4) 

(5) 

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Investigation (1)

Algebraic solution. Let the number of stones in each triangle be a,b,c,d.

Then $a + b = 62$)
$c + b = 48$) Add these 3 equations
$d + b = 54$)

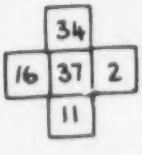
$\therefore\ a + c + d + 3b = 164$) subtract these
But we know $a + b + c + d = 100$) equations.
$\therefore\ 2b = 64$, $b = 32$

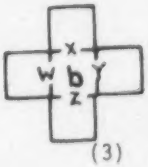
Now by substition $a = 30$, $c = 16$, $d = 22$.

In general if the number of stones is S then

$$b = \frac{X + Y + Z - S}{2}$$

(2) A similar approach can be used to find the value of the middle square, b.





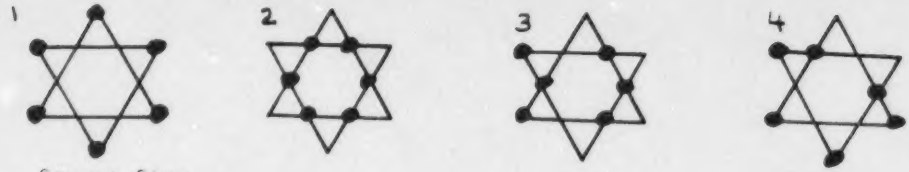
$$b = \frac{W + X + Y + Z - S}{3}$$

(3) In the pentagonal star puzzle the centre space holds $\frac{E - S}{4}$ where E is the sum of the numbers.
When setting these puzzles it is necessary to make the numbers sum to a number which is greater than S by some multiple of 4. It is also necessary that no number be less than $\frac{E - S}{4}$

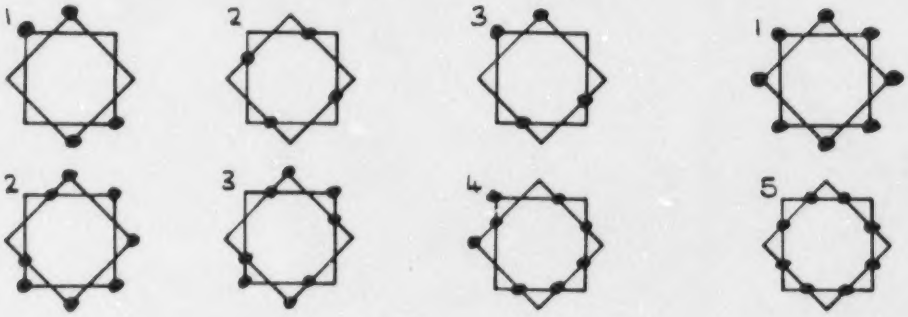
With the Hexagon Star puzzle, the sum of the numbers placed should be greater than S by a multiple of 5. No number should be less than $\frac{E - S}{5}$.

Page 17 Star puzzles - answers and notes

Triangle Star



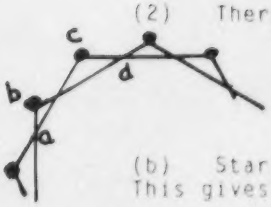
Square Star



Page 18 Pentagonal Star

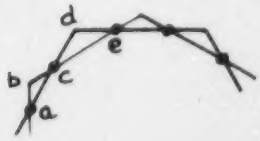
Investigation

(1) Puzzles with hexagons: 6 stones :- 3, 4, 5 or 6 each hexagon.
12 stones :- 6,7,8,9,10,11,12, each hexagon.



(2) There are two general methods of solution (a) Start with stones on the points of the star. This gives the fewest on each polygon. Slide b → a and c → d. This still leaves 2 stones on each line and increases the number on each polygon by 1.

(b) Start with stones on the points of intersection This gives the greatest number of stones on each star.



Jump a over c to d.
Jump e over c to b.
This decreases the number of stones on each polygon by 1.
This can be repeated.

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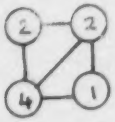
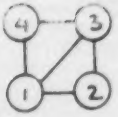
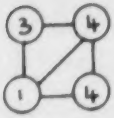
Square Puzzles - answers and notes.

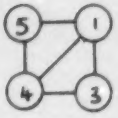
(1)

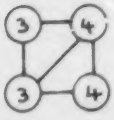
(2) 

(3)

(4) 

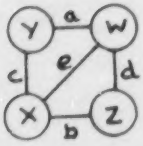
(5) 

(6) 

(7) 

Investigation:

(1) Add any pair of numbers on opposite sides of the square. In Puzzle 6, 9+4 = 13, So 13 stones are needed. In Puzzle 7, 7+5 = 12, So 12 stones are needed.



(2) General Method of Solution

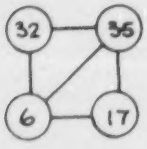
Consider the three circles that make up one half of the square XYW.
Number of stones in $\triangle XYW = \frac{a+c+e}{2}$

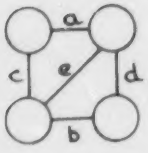
$\therefore Y = \frac{a+c+e}{2} - e = \frac{a+c-e}{2}$

Having calculated the number of stones in one circle, work around the square:

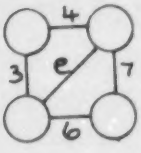
$W = a-Y$, $X = c-Y$

and $Z = b-X$.

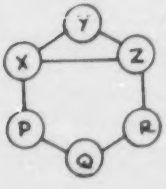




(3) a+c+e and b+e+d must be even.
Also e must be less than or equal to a+c and b+d.
Example with 10 stones.

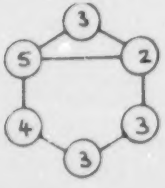


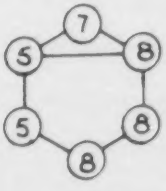
a+b = 10 and c+d = 10
To make a+c+e and b+d+e even then e must be odd. It must also be less than or equal to 7
$\therefore$ e could be 1,3,5 or 7.

(4) 

Solve for triangle XYZ first.
$X+Y+Z = (2+4+4) \div 2 = 5$
$X+Z = 4$ $\therefore Y = 5-4 = 1$
$\therefore X = 1$, $P = 4$, $Q = 1$, $R = 3$, $Z = 3$.

The other two solutions are





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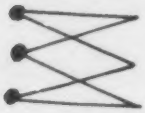
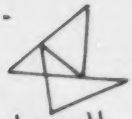
Polygrams - answers

Hexagrams

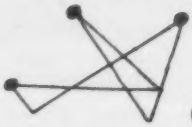
(1)

(2)

(3) 

(4) 

(5)

(6) 

(7)

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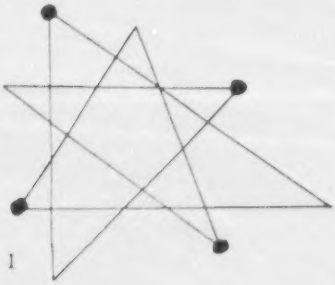
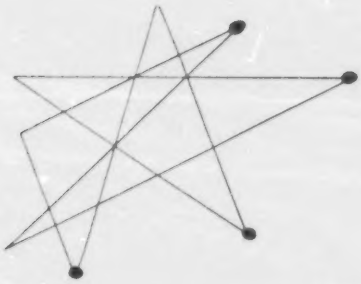
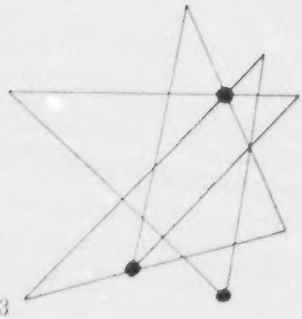
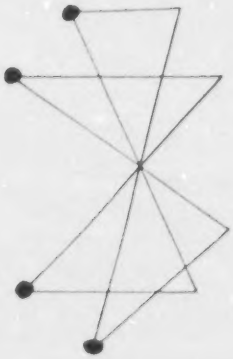
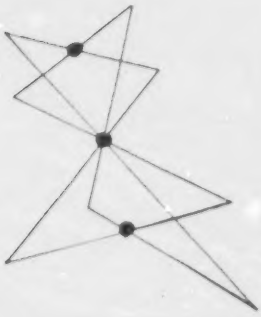
Octagrams

1 

2 

3 

4 

5 

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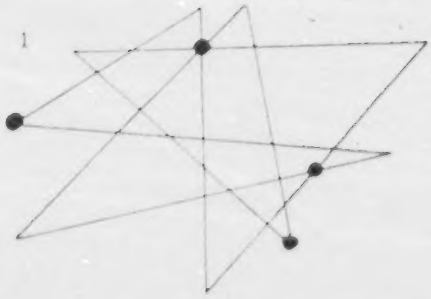
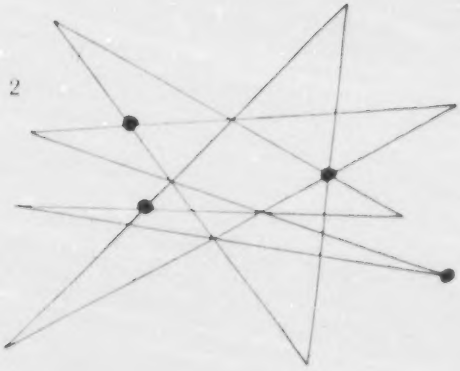
Nonograms

(1)

1 

2 

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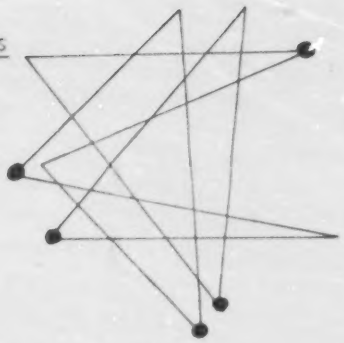
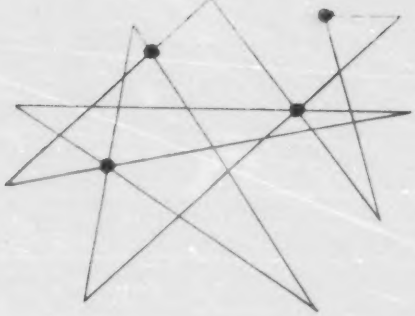
Decagrams

(1)



(2)



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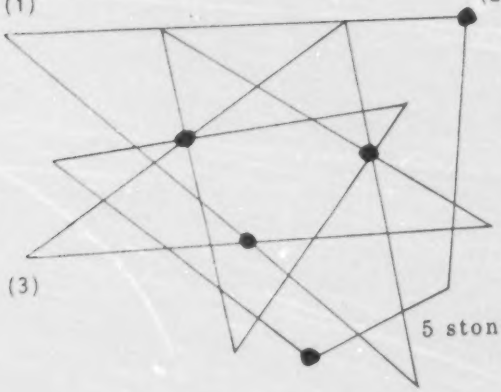
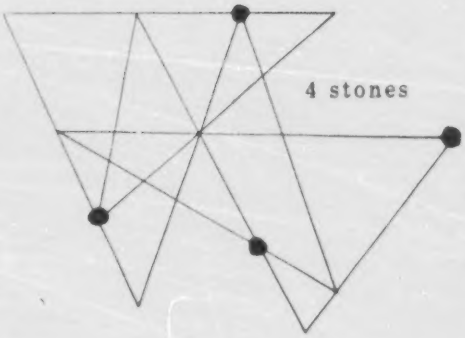
Harder Polygram Puzzles

(1)



(2)

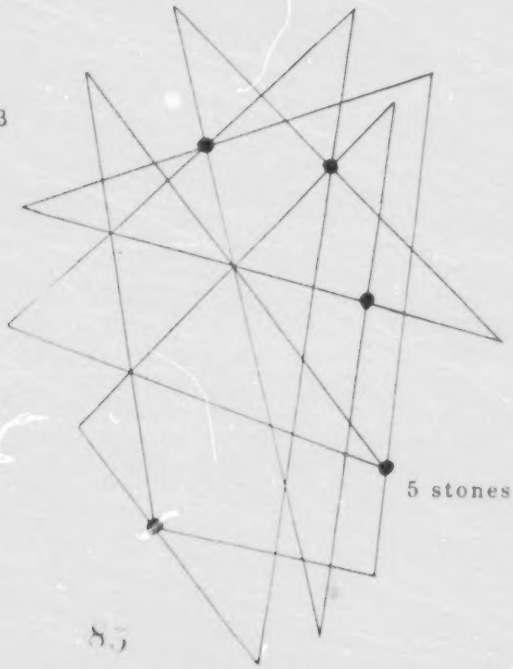


4 stones

(3)

5 stones

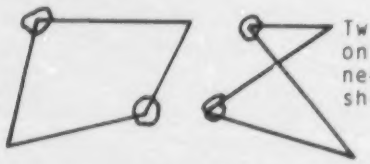
3



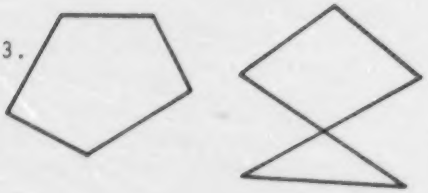
5 stones

Notes on Investigation

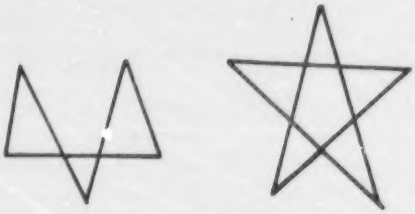
1. A trigram is simply a triangle. It is not possible to place stones at the vertices so that there is one on each line.

2. 

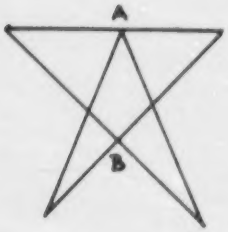
Two possibilities only. 2 stones needed at positions shown.

3. 

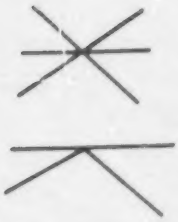
Vertices	How many Lines
5	2,2,2,2,2
6	2,2,2,2,2,2
7	2,2,2,2,2,2,2
10	all 2's



All these pentagrams make impossible puzzles. All the vertices are even (2 lines meet or pass through them). A stone placed at a vertice will be on 2 lines. There are 5, an odd number of lines ∴ no solution.



3 lines meet at A so now the puzzle will be possible. 5 lines are covered, 3 by A, 2 by B.



'odd' vertices

4. In general polygrams with 2 N lines will always be possible of solution using N stones.
If there are vertices where 3 lines meet then:

1. If there is just one it must avoided.

2. If there are two sharing a line they must be avoided.

3. If there are two which have no line in common both can be used and the number of stones becomes N-I.

The same tactics are applied where ever 'odd' vertices appear. They can be used in pairs as long as no line is shared.

If there are 'even' vertices where 4, 6 etc lines meet these can be used and reduce the number of stones needed.

Where the polygrams have 2 N + I sides they are impossible if all vertices are even.

1. If odd vertices are present then these make the puzzle possible.

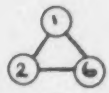
2. Odd numbers of odd vertices can be used provided they do not share straight lines.

Triangles with a difference - answers and notes

Puzzles 1

2

3

4 

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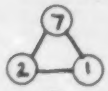
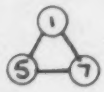
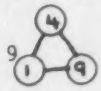
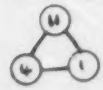
6 

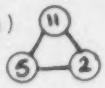
7

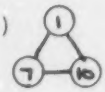
8 

9 

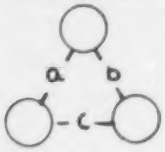
10

11 

12(a) 

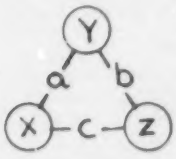
12(b) 

INVESTIGATION 1. There are 8 possible puzzles.
The values of a, b and c can be
0,7,7 : 6,5,1 : 4,0,4 : 3,2,5:
3,1,2 : 1,3,4 : 0,2,2 : 1,0,1.



2. A pair of the numbers between the circles sum to the third. If $a \ll b \ll c$ then $a + b = c$

3. The simplest method with small numbers is trial and error. Notice that the smallest number of stones will always be in one of the circles next to the biggest difference. A general method of solution is as follows:



If $a \ll b \ll c$
and S = Number of stones then

either $Z = \frac{S + b + c}{3}$, $x = \frac{S - a - c}{3}$

and $Y = \frac{S + a - b}{3}$ and $z \gg y \gg x$

or $Z = \frac{S - b - c}{3}$, $x = \frac{S + a + c}{3}$

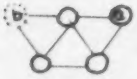
and $Y = \frac{S - a + b}{3}$ and $z \ll y \ll x$

4. There are two solutions when a, b, c, and s are all multiples of 3 and $b + c \ll S$

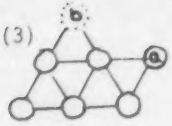
Change the shape - answers and notes

(1)

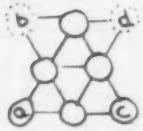
Move a to b.

(2) 

Move a to b.

(3) 

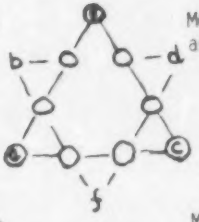
Move a to b.

(4) 

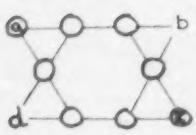
Move a to b and c to d.

(5)

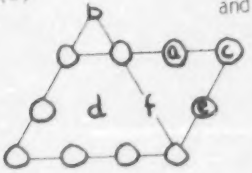
Move a to b and c to d.

(6) 

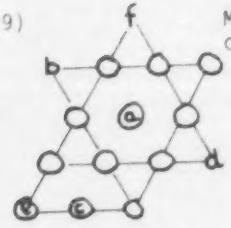
Move a to b, c to d. and e to f.

(7) 

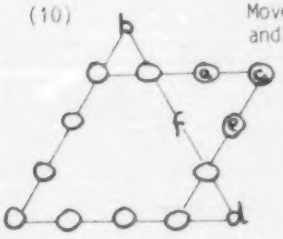
Move a to b and c to d.

(8) 

Move a to b, c to d. and e to f.

(9) 

Move a to b, c to d and e to f.

(10) 

Move a to b, c to d, and e to f.

Investigation

(1) Move 1 stone.

(2) Move 2 stones.

(3) With a 16 square move 4 stones. If N^2 is the number of stones then the number to be moved, M, is given by

$$M = \frac{N^2}{4} \text{ if } N^2 \text{ is even.}$$

$$M = \frac{N^2 - 1}{4} \text{ if } N^2 \text{ is odd.}$$

Pentagonal Puzzles - Answers

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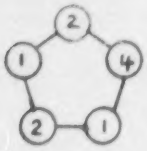
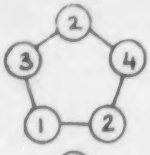
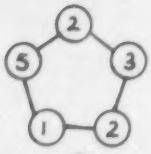
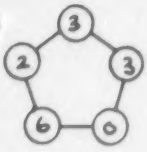
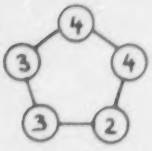
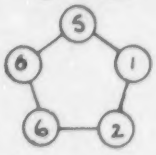
(1) 

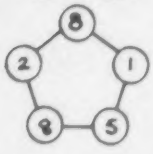
(2) 

(3) 

(4) 

(5) 

(6) 

(7) 

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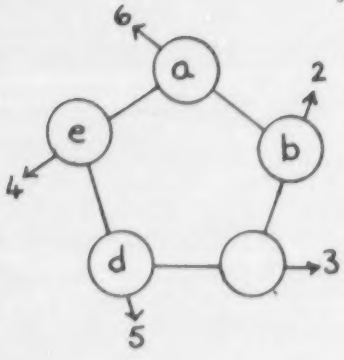
Notes

1. No, 1,2,3,4, and 5 add to 15 an odd number. The totals along the sides when summed is equal to twice the number of stones so this sum must be even.

2. Yes, for example 2,3,4,5,6. These sum to 20 so 10 stones will be used. In general only 5 consecutive numbers starting with an even number make an even sum and so will be possible.

3. No. Using 2,3,4,5,6 as an example This is not possible. 6 stones will have to be in a and b. 5 stones will will have to be in d and e:



$a + b = 6$
$e + d = 5$

but the total number of stones is 10 and $11 > 10$ $\therefore$ no solution (unless we introduce negative stones or holes!)

However if we consider the general case:
Let the first number in the series be 2N

So 2N, 2N + 1, 2N + 2, 2N + 3, 2N + 4 are the numbers around the pentagon.

Number of stones = $(2N + 2N + 1 \ldots 2N + 4) \div 2 = \frac{10N + 10}{2} = 5N + 5$

Highest values in series are 2N + 4, 2N + 3.

If these were placed on non intersecting sides

they would involve $(2N + 4) + (2N + 3) = 4N + 7$

Now $4N + 7 \ll 5N + 5$ for a solution

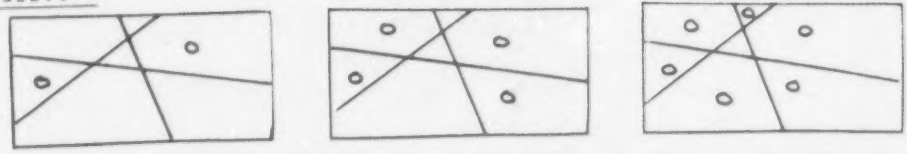
$\therefore 2 \ll N$

So for all other series than the one orginally considered (where n = 1) it will not matter where the numbers are placed

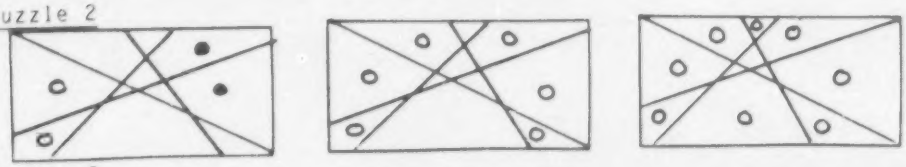
eg. N = 3 Series is 6,7,8,9,10
Number of stones = 20
Highest pair = 9 + 10 = 19

Barriers - answers and notes

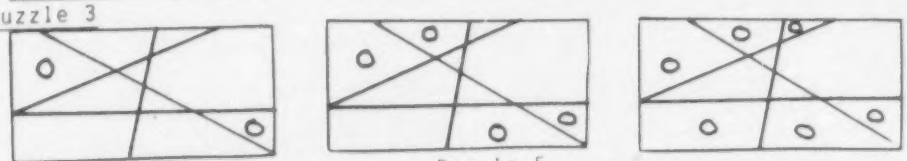
Puzzle 1



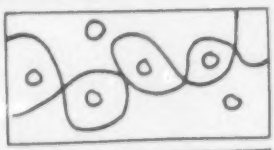
Puzzle 2



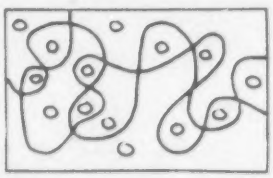
Puzzle 3



Puzzle 4



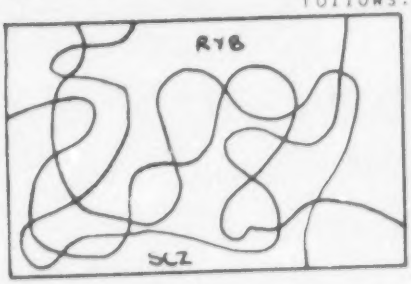
Puzzle 5



Puzzle 6

INVESTIGATION 1. On puzzle 7 we can mark each space as follows:

above the red line, R : below, S.
above the yellow line, Y : below, Z.
above the brown line, B : below, C.



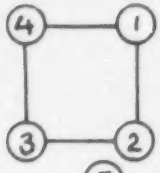
A stone placed in a RYB space will be matched with one SZC, one in RYC by SZB, one in RZC by SYB etc. In this way it is possible to place 16 stones.

2. In the two curved line puzzles the number of stones S is related to the number of intersections, I, as follows.

If I is odd, S = I + 3.
If I is even, S = I + 2.

TRAILS - ANSWERS AND NOTES

(1)



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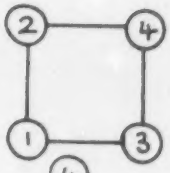

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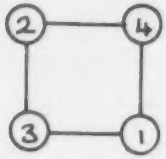

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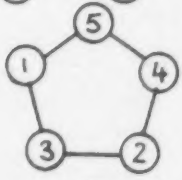

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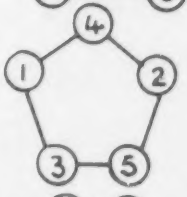

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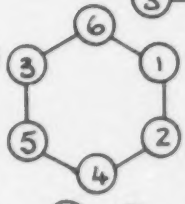

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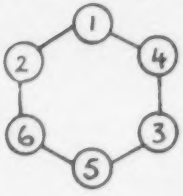

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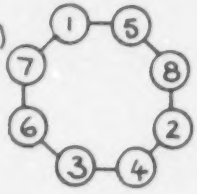

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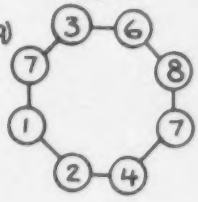

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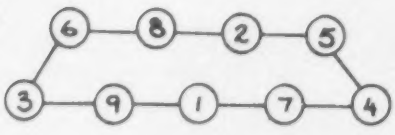

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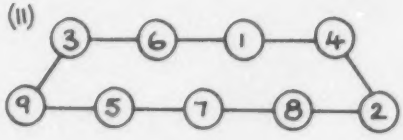

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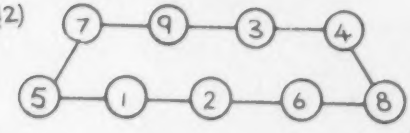
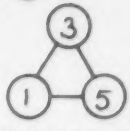


INVESTIGATION

1.



2. Solving by trial and error is inefficient when large numbers are involved. It is best to use algebra. Let the unknown be x. Then in the first problem

$3x + 2 - 4 = x \quad \therefore \; 2x = 2$

and $x = 1$

If the triangles produce an equation in which the x is cancelled out then the solution is not unique.

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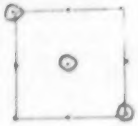
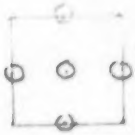
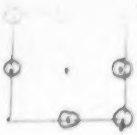
Dots - answers and notes

9 dots

Puzzle 1 

Puzzle 2 

Puzzle 3 

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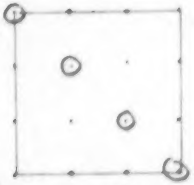
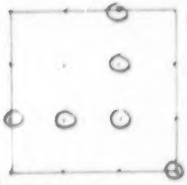
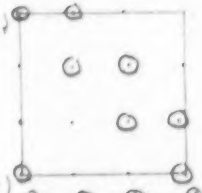
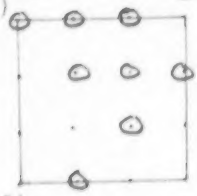
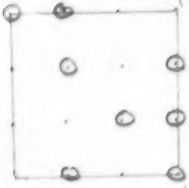
16 dots

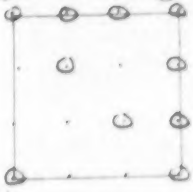
(4) 

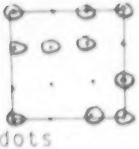
(5) 

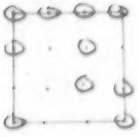
(6) 

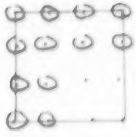
(7) 

(8) 

(9) 

(10) 

(11) 

(12) 

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Investigation - 25 dots

1. It is possible to make rows and columns odd using 5,7,9,11,13,15,17,21 and 25 stones. It is rather surprising that 19 and 23 are not possible. (On the 16 dot puzzle you will find that 11 and 13 stones can not be arranged to give odd lines).

2. The following numbers make even rows and columns: 10, 12,14,16,18,20. It is not possible using 22 or 24 (on the 16 dot puzzle 14 is impossible).

3. There is no number of stones which makes this possible. There are 5 rows. If each has an odd number then the sum of 5 odd numbers must be odd. So odd numbers of stones must be used to create odd lines. However an odd number of stones can not be divided into 5 even numbers.

Starting with an even number of stones and a 36 dot puzzle there are several solutions. In general, where the number of dots is N^2 there will be such solutions for all even values of $N > 2$ and one solution will be to use $2N$ stones.

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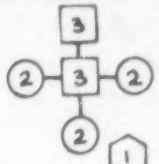
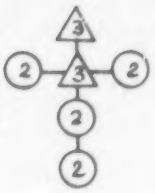
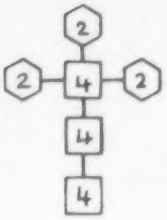
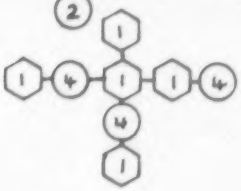
Simon's Puzzles - answers

(1) 

(2)

(3) 

(4) 

(5) 

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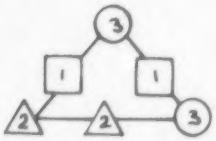
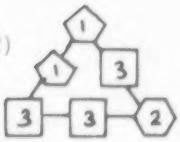
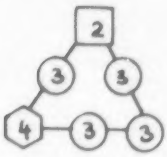
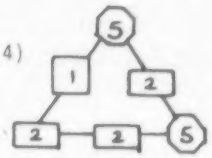
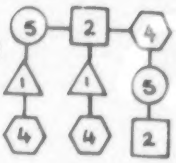
(1) 

(2) 

(3) 

(4) 

(5) 

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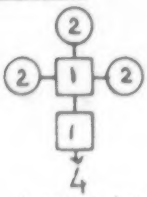
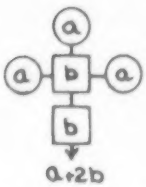
Investigation

(1) 

(2) 

The total number of stones, S = 3a+2b
Substituting in integer values for a and b, we find S can have any value but 1.

(3) Putting letters to stand for the number of stones we can get SIMultaneous equatiONS for each of Simon's Puzzles.

In this example 2a+b = 63 .....(1)

a+2b = 100 .....(2)

Multiplying (1) by 2 and subtracting (2) gives

a = 26

Substituting gives b = 11

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Pile Up - answers and notes

Puzzle (1) M to N, N to Dump, O to Dump.

(2) M to N, N to Dump, L to M, M to N, N to O, O to Dump.

(3) K TO L, M to N, O to Dump, N to Dump, L to N, N to Dump.

Page 52 (4) M to N, N to Dump, O to Dump, J to K, K to M, L to M, M to Dump.

(5) I to J, J to L, K to L, L to Dump, M to N, N to Dump, O to Dump.

(6) J to K, K to M, M to Dump, O to Dump, N to O, O to Dump, H to I, I to K, K to M, L to M, M to Dump.

(7) As for (4) then G to H, H to J, I to J, J to M, M to Dump

(8) As for 3 then J to K, H to I, I to K, F to G, G to I, I to K, K to Dump.

(9) As for 5, then E to F, F to H, G to H, H to L, L to Dump.

(10) As for 5, then A to B, B to D, C to D, D to H, E to F, F to H, G to H, H to dump.

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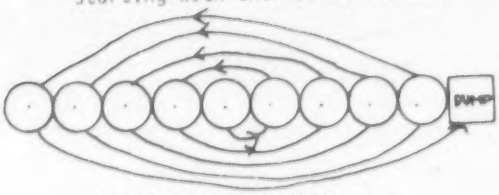
INVESTIGATION: (1) 1 stone - 1 move, 3 stones, - 3 moves, 7 stones - 7 moves, 15 stones - 15 moves.

The next will be 31 stones.

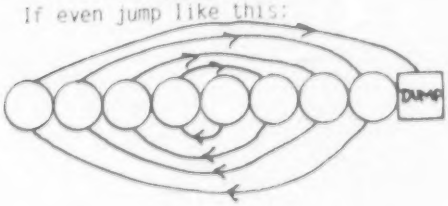
The series is 1,3,7,15,31, $2^N - 1$.

(2) The number of moves equals the number of stones for all puzzles.

(3) If the number of stones is odd jump like this, starting with the Middle stone:



If even jump like this:



Cover Up - answers

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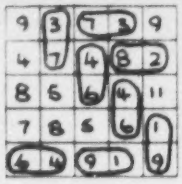
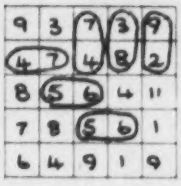
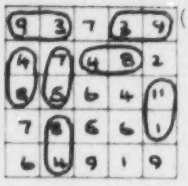
Puzzle 1 (a)



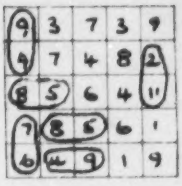
(b)



(c)

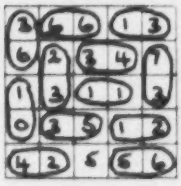


(d)



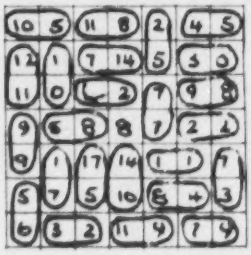
Puzzle 2.

5 is uncovered



Puzzle 3. *Page 55*

8 in the middle is uncovered.
It is easier to work this puzzle backwards: cover 24 first.



Hexapla - answers.

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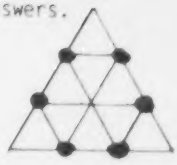
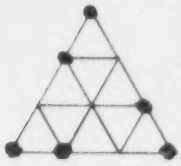
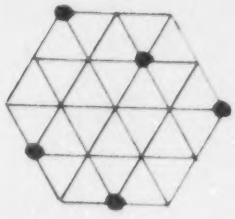
Puzzle 1



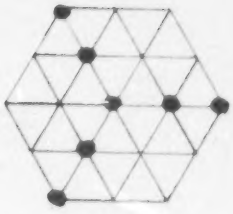
2



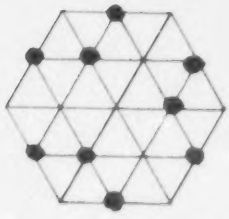
3



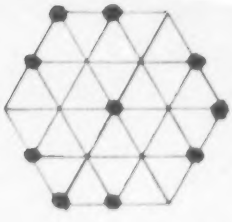
4



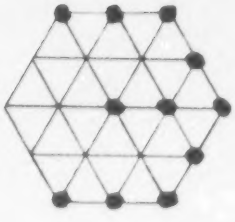
5



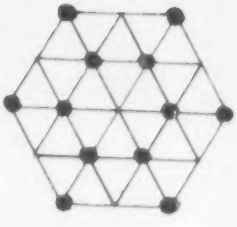
6



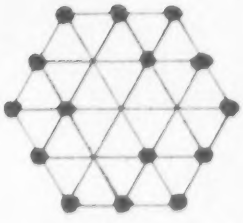
7



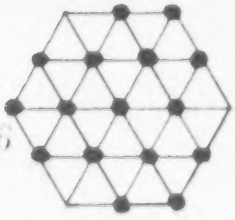
8



9



10



Squares - answers.

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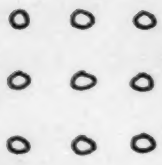
Puzzle (1)

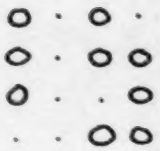
(2)

(3)

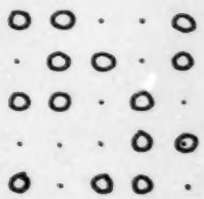


(4)

(5) 10 stones



(6) 15 stones



6,10,15 triangle numbers! The number of stones for 1 dot is 1 and for 4 dots is 3. This suggests that the next, 36 dots, may take 21 stones. But why?

Factors - answers

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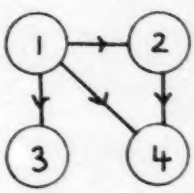
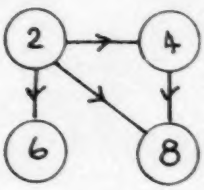
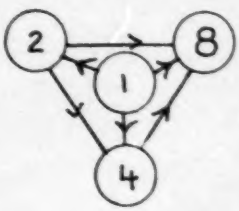
Puzzle 1



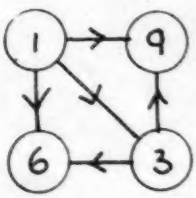
2



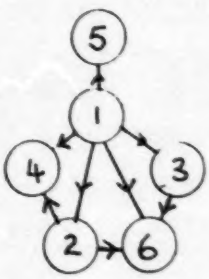
3



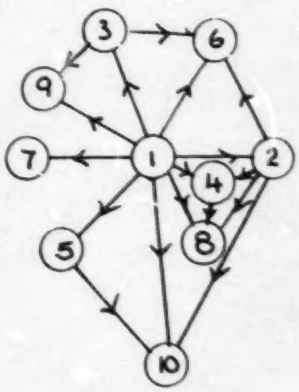
4



5



6



Lines - answers

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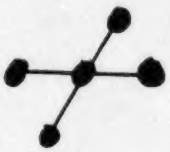
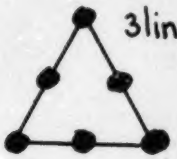
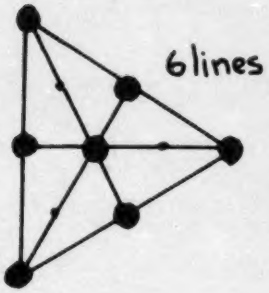
5 stones



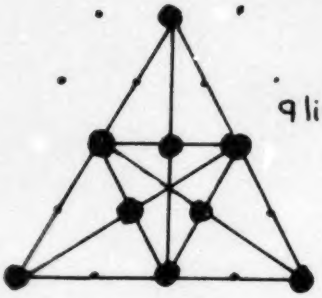
6 stones



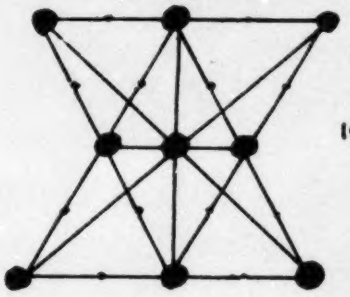
7 stones



9 stones



9 stones



A trick

Place the last stone on top of the one in the centre.

Thirds - answers

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2 move puzzles

(1) 9,9,9 → 6,9,12 → 6,6,15
(2) 9,9,9 → 9,6,12 → 11,4,12
(3) 9,9,9 → 12,9,6 → 8,13,6
(4) 9,9,9 → 12,9,6 → 8,9,10
(5) 9,9,9 → 12,9,6 → 14,9,4

3 move puzzles

(1) 9,9,9 → 12,9,6 → 15,6,6, → 17,6,4
(2) 9,9,9 → 12,9,6 → 15,6,6 → 15,8,4
(3) 9,9,9 → 9,6,12 → 9,10,8 → 5,10,11
(4) 9,9,9 → 12,9,6 → 14,9,4 → 14,7,6
(5) 9,9,9 → 6,12,9 → 6,8,13 → 4,10,13
(6) 9,9,9 → 9,6,12 → 11,4,12 → 11,8,8

4 move puzzles

(1) 9,9,9 → 9,6,12 → 6,6,15 → 6,14,7 → 4,4,19
(2) 9,9,9 → 12,9,6 → 14,9,4 → 14,7,6 → 16,7,4

Stone Paths - Answers

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Puzzle 1. 1,3,5,6,2 Puzzle 2. 1,2,6,9,10,6,1.

Puzzle 3

Highest total is 54: 2,3,7,4,5,9,4,6,8,5,1

MAZES - Answers

Page 63

Maze 1. 2 to 1 (B), to 2 (up, right), to 2 (down left), to 1 (across), to finish.

Maze 2. 2 → to 1, ↙ to 3, → to 4, ↖ to 2, → to 1, ↙ to 3, ↘ to 1, ↙ to 3, ↗ to 1, ↘ to finish.